COMMENTS ON VISCOSITY, CRYSTAL SETTLING, 
AND CONVECTION IN GRANITIC MAGMAS*

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ABSTRACT. The effect of H$_2$O on the viscosity of silicic silicate melts is now known. Accordingly some limitations are placed on the fluid properties of granite magmas. Most granite melts are expected to have viscosities between 10$^6$ and 10$^8$ poises at temperatures at or above the liquidus. At temperatures down to 100°C or more below the liquidus it is expected that the melts would behave virtually as Newtonian fluids. Hydrostatic pressure, at least to about 7 kilobars has little effect on the fluidity. The effective viscosity of magma containing suspended crystals is predicted to increase by less than an order of magnitude over that of the melt fraction for amounts up to nearly 50 percent crystals by volume. Bubbles of a gas phase are also predicted to increase the effective viscosity and by a similar amount.

Crystal settling is a factor of possible importance in crystallization of granitic magmas and under some plutonic conditions could be measured in kilometers. Forced convection of granitic magma, under certain conditions, can result in virtually instantaneous formation of dikes. Natural convection is possible in stock-like bodies of magma less than 100 meters in diameter if connection with an underlying magma reservoir exists, and natural convection must be reckoned with in any large granitic pluton, at least in the earlier stages of crystallization.

INTRODUCTION

Recent data on viscosities of silicate–H$_2$O melts approximating granites in composition make possible some limited generalizations concerning magmatic mobility and effectiveness of crystal settling during emplacement and crystallization of silicic igneous rocks. A number of results can be stated in quantitative terms which however require numerous qualifications when applied to conditions in the Earth's crust. Despite these misgivings some of the results obtained may be of interest to geologists pursuing problems of the reconstruction of physical mechanisms active during the evolution of at least some granitic rocks. Most concepts involved are simple, but because of interdependent factors the resultant problem is intricate, even for geologically clear-cut boundary conditions.

The discussion will state briefly the basis for viscosities in silicate–H$_2$O liquids and illustrate their implications in general terms with regard to settling of various minerals and simpler aspects of fluid flow. These topics require consideration of problems such as shape factors in crystal settling, rates of crystallization and maximum crystal sizes, viscosities of suspensions, and possibilities of convection.

VISCOITIES OF GRANITIC MELTS

The principal data considered are those of Shaw (1963) and Friedman, Long, and Smith (1963) which were generalized somewhat by Shaw (see fig. 1). Burnham (1963) also gives some data at high H$_2$O content, and they are in fair agreement with the predictions of figure 1. The bulk composition of the silicate represented by the viscosity isotherms in figure 1 is close to compositions in the system Al$_2$O$_3$–Or–SiO$_2$–H$_2$O, and thus the dashed curve, which is the superposition of the "ternary minimum," gives roughly the lower temperature (and higher viscosity) limits of the field in which any granitic liquids can be thermodynamically stable. Above this line is the region of supercooled or meta-
stable liquids. Thus the viscosities of stable liquids of ternary minimum composition, coexisting with an H–O-rich gas phase, are expected to lie within relatively narrow limits of the viscosity \(8 > \log_{10} \eta > 6\) over a wide range of \(H_2O\) contents exceeding about 1 percent by weight. The total fluid pressure, of course, varies along the dashed curve from vanishing at vanishing \(H_2O\) content to about 4000 bars at 10 percent \(H_2O\) by weight.

Although no detailed discussion will be given of the chemical basis of these results for the viscosities of silicate–\(H_2O\) liquids, one aspect will be demonstrated which may give some insight. Figure 2 shows viscosities as a function of temperature for various constant compositions expressed as the atomic ratio \(R = (Si+Al)/O\). Note that for a given ratio, curves based on viscosities taken from figure 1 correlate well with viscosities for anhydrous silicate melts of more mafic compositions determined by Enler and Winkler (1957). The range of validity of figure 1 is thought to extend only to about
Fig. 2. Correlation of viscosities of silicate melts of diverse composition in terms of the atomic ratio $R = (\text{Si} + \text{Al})/\text{O}$. The dashed curves, labeled in weight percent, are based on viscosities taken from figure 1, where, to a good approximation, $R$ is given by the relation $R = (100-m)/(200-m)$, where $m$ is mole percent $\text{H}_2\text{O}$. The solid curves are for anhydrous melts from Euler and Winkler (1957, fig. 2): (a)—andesite; (t)—tephrite; (b)—olivine basalt. Curve (b) is analogous to $\text{H}_2\text{O}$ contents beyond the range of validity of figure 1 and is shown only for contrast. The dashed curves at 1 and 2 percent $\text{H}_2\text{O}$ by weight also are shown only for contrast inasmuch as the data of Euler and Winkler do not extend to analogous compositions.

10 percent $\text{H}_2\text{O}$ by weight, and consequently the correlation is not expected to be valid at ratios less than about $R = 0.42$, and the divergent trend of the curve (b) for olivine basalt bears this out. Within the range of validity, however, the correlation suggests that the principal effect of composition on viscosities of silicate melts, whatever the oxide components in addition to silica, is the extent to which the silicon-oxygen framework is disrupted.

Obviously the above correlation is an oversimplification in that it is known that melts of the same ratio $(\text{Si} + \text{Al})/\text{O}$ can have different viscosities; compare viscosities for the various feldspar compositions and silica all with a ratio $R = 0.50$ (Birch, Schairer, and Spicer, 1942, table 10-1). Melts in the diorite-granite range of compositions, however, are mixtures in which the contrasting effects of the individual feldspar and silica components are decreased, and the result is that the curves illustrated in figure 2 represent a sort of average behavior for melts of intermediate to silicic compositions. Thus rough predictions of viscosities in this range are possible by means of the curves in figure 1 by converting to the appropriate value of $R$. To a good approximation $R$ is related to mole percent $\text{H}_2\text{O}$, as given in figure 1, by $R = (100-m)/(200-m)$, where $m$ is mole percent $\text{H}_2\text{O}$. This assumes the anhydrous composition to have the ratio $R = 0.50$ (the actual ratio of anhydrous glasses in the experiment by Shaw, 1963, is $R = 0.498$).
It has been demonstrated (for example, see Bockris, MacKenzie, and Kitchener, 1955; Shaw, 1963) that silicate liquids behave as Newtonian fluids. In other words no peculiar behavior is to be expected as a result of widely differing rates of shear. This result greatly simplifies application of the data to concepts of fluid mechanics as long as we are dealing with temperatures near and above the liquidus. It is well known, of course, that glasses behave much like elastic solids at room temperature, and consequently the above conclusion becomes invalid at some temperature below the liquidus. The temperature at which elastic effects become notable must be determined for a particular glass, but some generalizations can be made. Every glass when heated to or cooled from the liquid range passes through an interval of temperature known as the transformation or transition range where there are marked time-dependent changes in its physical properties. This occurs at temperatures considerably below the liquidus where the viscosity reaches such high values, on cooling, that attainment of internal equilibrium is greatly slowed. At lower temperature the thermodynamic state of a glass depends on the thermal history above and within the transformation range. When a liquid is cooled below liquidus temperatures it continues to behave as a liquid, although a metastable liquid, with no abrupt changes in its properties until the transformation range is reached. The lowest temperature at which a glass can approach internal equilibrium within a short time (minutes) is known as the annealing point and is taken, for practical purposes, to be the temperature at which the viscosity is about $10^{13}$ poises (see Weyl, 1951, p. 302). Note that the range of viscosities considered in figure 1 is far below $10^{13}$ poises. Thus hydrous melts probably exhibit Newtonian behavior even for undercooling of more than 100°C, at least for H$_2$O contents exceeding a few tenths of a percent. The annealing point for a completely anhydrous glass would presumably be somewhat nearer the liquidus than for hydrous glasses, judging from the trend of the dashed curve in figure 1.

There is considerable literature concerning glass transformations, but two papers are of such clarity that they are of particular interest (Jones and Simon, 1949; Condon, 1954). Theoretical work concerning the properties of glasses also has been done by many, but the concepts introduced by A. Q. Tool have been particularly successful (Tool, 1946; Condon, 1954).

Murase (1958) recently reported on some experiments with obsidian which bear on the glass transition in natural glasses of salic composition. He performed bending experiments on rectangular strips of glass at controlled temperatures, and the results permitted computation of both the elastic and viscous contributions to the deformation. He found that Young's modulus was nearly constant up to 750°C and then fell off abruptly at slightly higher temperatures. This suggests that the transition range and annealing point are near this temperature, and in support of this conclusion it appears from his results that the viscosity at 750°C is about $10^{13}$ poises. The vapor pressure of H$_2$O apparently was not controlled in these experiments, and presumably the results are for nearly anhydrous glass.

Murase (1962) reports much additional work of the same kind on a number of different volcanic rocks. The results were similar but with complications
due to the heterogeneous nature of the rocks. In this same paper Murase (1962, p. 534-553) discusses the effect of composition on the viscosity based on Buerger's (1948) concept of bridge density. On this basis he predicts the effect of H₂O on the viscosity, and his results resemble those of Shaw (1963) which were based on similar principles, stated differently, but which also had greater experimental control. Murase's viscosity isotherms, however, are of considerably different form from those of Shaw, and the viscosities depart markedly from values reported by Shaw.

An additional result of the data given by Shaw (1963) is that hydrostatic pressure appears to have a small effect on the viscosity at constant temperature and composition. This conclusion is supported by the agreement of data by Burnham (1963) at pressures to 7.4 kilobars. Thus there is no reason to propose, as some geologists have in the past, drastically different models for fluid behavior of homogeneous melts under plutonic and near surface conditions, except as may be required by composition (for example, H₂O content) and temperature differences.

The petrologist, now enabled to estimate viscosities of magmas containing H₂O, would undoubtedly desire to have some basis to guide his intuition or mental perception of what these viscosities may mean when viewed in reference to certain familiar kinds of fluid behavior. Unfortunately the sort of fluid behavior that must be dealt with in order to reconstruct the mechanical history of magmatic rocks is difficult to perceive on the basis of simple analogies. The physical meaning of the viscosity will be viewed here in terms of the kind and extent of fluid motion, or differential solid-fluid motion, under specified boundary conditions. Discussion of a particular geologic process in detail requires an appropriate dimensional or scale model analysis (see Hubbert, 1937; Ramberg, 1963), whereas the present paper comments only on some general relationships.

CRystal SETTLING

Settling rates of individual crystals of fixed size can be predicted readily. Unfortunately the results do not tell us a great deal about movement of crystals in magmas except for such a simple case as estimating the extent of movement of scattered phenocrysts in a batch of magma intruded or extruded into a container of simple form for which the subsequent cooling history is known. In general we need to know several factors which are interrelated. That is, we need to know the kind and size of crystals as a function of time, the number of crystals of various changing sizes, the effect of changing composition, temperature, and volume concentration of crystals on the bulk viscosity of the magma, together with a knowledge of temperatures as a function of time. This rather formidable list is needed for the simple case where there is no mass motion of the liquid except as a result of displacement by settling crystals. If natural or forced convection occurs, as seems likely for some conditions, the prediction of differential movement of crystals and liquid is greatly complicated.

General quantitative solutions to this problem in analytical form obviously would be of extreme intricacy, and no attempts have been made, to the author's knowledge, to derive such solutions in existing literature on kinetics and fluid
Table 1
Symbols

$t$ —time, sec.

$\eta$ —Coefficient of viscosity, gm cm$^{-1}$ sec$^{-1}$, or dyne sec cm$^{-2}$ (poise).

$\eta_s$ —Coefficient of viscosity of a suspension where $\eta_0$ is that of the liquid fraction.

$\eta' = \frac{\eta_s}{\eta_0}$

$\nu$ —Kinematic viscosity ($= \eta/\rho$), cm$^2$ sec$^{-1}$.

$g$ —Acceleration of gravity, 980 cm sec$^{-2}$.

$v$ —Velocity, cm sec$^{-1}$.

$v_n$ —Velocity of sphere having volume of a non-spherical particle.

$v_s$ —Settling velocity of a suspension.

$v_n$ —Settling rate at infinite dilution.

$\rho$ —Density ($\rho_1$ — density of liquid; $\Delta \rho$ — density contrast between solid and liquid), gm cm$^{-3}$.

$r$ —Radius of particle or radial distance in equation for line source of heat, cm.

d —Distance traveled by settling particle, cm.

$a$ —Radius of cylindrical conduit, cm.

$s_1$, $s_2$ —Sides of rectangular cross section of a conduit, cm.

$l$ —Length of conduit, cm.

$r_h$ —Hydraulic radius; ratio of area of cross section to length of wetted perimeter.

$\Psi$ —Function of $s_1$, $s_2$.

$\phi$ —Volume fraction of suspended solid in solid-liquid mixture.

$K$ —Shape factor ($= v_n/v$).

$Q$ —Volume rate of flow, cm$^3$ sec$^{-1}$.

$Q_h$ —Total rate of convective heat transfer, cal sec$^{-1}$.

$P$ —Pressure drop in forced convection.

$T$ —Temperature ($T_L$ — temperature at liquidus), deg.

$(T_0 - T_1)$ —Temperature gradient from axis to margin of conduit.

$\xi$ —Rate of crystal growth on some specified basis (linear, radial, etcetera), cm sec$^{-1}$.

$c$ —Constant rate of radial growth of a spherical particle, cm sec$^{-1}$.

$\xi$ —Rate of crystal nucleation, cm$^3$ sec$^{-1}$.

$k_1$ —Empirical constant in equation (9) for linear rate of crystal growth.

$\eta_a$ —rate at which heat is released from a line source, cal cm$^{-2}$ sec$^{-1}$.

$\beta$ —Variable of integration in equation (20) for heat flow from a line source.

$k$ —Thermal conductivity, cal cm$^{-2}$ sec$^{-1}$ deg$^{-1}$.

$\kappa$ —Thermal diffusivity ($= k/\rho C_p$), cm$^2$ sec$^{-1}$.

$C_p$ —Heat capacity at constant pressure.

$\alpha$ —Coefficient of cubic expansion, deg$^{-1}$.

Non-dimensional numbers:

Reynolds number: $Re = 2 \rho_1 r v / \eta$.

NOTE: This is the particle Reynolds number for $r$ the particle radius and $v$ the particle velocity. For the case of fluid flow in channels the Reynolds number takes the same form with $r$ replaced by $2a$ ($= a$, for the special case of a cylindrical channel), with $v$ the average rate of flow.

Praudil number: $\sigma = v / \kappa = \eta C_p k$

Nusselt number based on radius: $Nu = \frac{Q_h}{2\pi k (T_0 - T_1)}$

Modified Grashof number based on radius: $Gr = \frac{\alpha a^3 (T_0 - T_1)}{\nu k}$

Non-dimensional temperature function for laminar flow:

$T_n = \frac{\alpha a^4 (T_0 - T_1)}{\nu k l} = \frac{a}{l} Gr$
mechanics. Yet all of the variables mentioned are implicit in attempts to describe the history of a rock of magmatic origin.

**Settling of individual particles in a homogeneous fluid.**—The settling velocity of a single spherical particle in a viscous fluid of great extent, by comparison with the diameter of the sphere, is described by Stokes' law

\[ v = \frac{2g\tau^2(\Delta \rho)}{9\eta} \tag{1} \]

where \( \Delta \rho \) is the difference between the density of the particle and the fluid (all symbols and dimensions are condensed in Table 1). The effect of the vertical walls of a container on the velocity is, for our purposes, unimportant at distances greater than about 10r. Also, in order that the behavior may be described accurately by Stokes' law, the viscosity must take large values compared with the product of the size and settling velocity of the particle. This criterion is generally described in terms of a dimensionless ratio called the particle Reynolds number given by \( R_e = 2\rho_v v / \eta \). The theoretical significance of the Reynolds number need not concern us since the number can be determined experimentally and can be compared with deviations from settling rates predicted by Stokes' law. The added effect of deviations from a spherical shape requires either special theoretical consideration or, more simply, experimental demonstration. Numerous experimental studies of various aspects of settling velocities exist, but we will refer only to a few in the course of this discussion.

The study by McNown and Malaika (1950) is especially informative in several respects. These authors consider the various relationships between Reynolds number, effects of container walls, shape, and orientation for various types of symmetric particles. Their results are presented in terms of graphs of a dimensionless shape factor, \( K \), versus values of the Reynolds number where \( K = v_n / v \), with \( v \) the settling velocity of the particle and \( v_n \) the settling velocity of a sphere having the same volume as the particle. McNown and Malaika find the range of validity of Stokes' law to extend to about \( R_e = 0.05 \) based on the nominal radius (the radius of a sphere having the same volume as the actual particle in question). We can readily show the maximum sized particles, in terms of the nominal radius, at the limit of the Stokes range by the relationship

\[ r = \left( \frac{4g\rho_1 \Delta \rho}{9\tau^2 R_e} \right)^{1/3} \tag{2} \]

Some values of the maximum nominal radius are shown in Table 2 for various viscosities and specified densities. Variations in the density contrast (\( \Delta \rho \)) between 0.1 and 1.0 do not change these limits greatly, and this range encompasses most density contrasts of interest with regard to crystal settling in granitic magmas. These results merely show that Stokes' law accurately describes rates of settling of individual particles for viscosities of interest in this paper.

On the basis of the results found by McNown and Malaika, rates of crystal settling have been plotted in figure 3 with suitable adjustments for shape, treating biotite as thin plates and hornblende as rods, oriented perpendicular to the motion, and other minerals as equant particles. Obviously, if biotites and hornblendes are more nearly equant their curves will shift toward that for
pyroxene because the densities are similar. The curves for crystal settling in basalt given by Hess (1960, fig. 30) are shown for contrast. All these curves can be adjusted for different viscosities by shifting them parallel to the abscissa in inverse proportion to the viscosity. The curves in figure 3 apparently suggest that extensive settling could occur in granitic magmas even for moderately small crystals if times of thousands of years are considered, as is probably the case for even small granitic plutons. For example, crystals of plagioclase 2 millimeters in diameter would settle about 500 meters in a thousand years or about 3 kilometers in ten thousand years.

We could make distances of crystal settling more applicable to magmatic conditions if we could make allowance for crystal growth during the time settling occurs. In the absence of quantitative information on the interrelations of growth rates, nucleation rates, change of liquid composition, and cooling history, we can do this only in an arbitrary manner. Nevertheless it may be of interest to see what kind of effect the changing size can have on distances a growing particle might travel. The simplest approximation is to take various constant growth rates (dr/dt) chosen to allow crystals of certain sizes to be attained in a given length of time. With the radius known as a function of time the distance travelled is obtained simply by integrating equation (1).

\[ d = \frac{2g\rho c^2 \Delta \rho t^n}{27\eta} \]  

(3),

where \( d \) is the distance, \( t \) is time, and \( c \) is the growth rate constant. Results are shown in figure 4 for various growth rates at a viscosity of \( 10^6 \) poises. The dashed lines represent times and distances where crystals reached 1 millimeter and 1 centimeter radii. The dotted lines represent distances travelled by particles with fixed radii of 0.1 and 1 centimeter respectively. Note that a crystal of fixed size settles only about three times as far as one that has continuously grown to this size at a constant rate during the same time of settling.

McNown and Malaika (1950) made the interesting observation that within the Stokes range \( (R_e < 0.05) \) particles fell in the orientation in which they

*Table 2*

Approximate maximum values of the nominal radius corresponding to the upper limit of the Stokes range, taken at a Reynolds number of \( R_e = 0.05 \), for \( \rho_1 = 2.30 \) g/cc, \( \Delta \rho = 0.3 \) (alkali feldspar) at various values of the viscosity. The density of the liquid is based on the density of a homogeneously hydrated obsidian containing about 6 percent H₂O by weight.

<table>
<thead>
<tr>
<th>( \log_{10} \eta )</th>
<th>( r ) (cm)</th>
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<tr>
<td>2</td>
<td>1.1</td>
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<tr>
<td>3</td>
<td>5.5</td>
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<tr>
<td>4</td>
<td>26</td>
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<tr>
<td>5</td>
<td>( 1.1 \times 10^6 )</td>
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<tr>
<td>6</td>
<td>( 5.5 \times 10^6 )</td>
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<tr>
<td>7</td>
<td>( 2.6 \times 10^6 )</td>
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<tr>
<td>8</td>
<td>( 1.1 \times 10^6 )</td>
</tr>
<tr>
<td>9</td>
<td>( 5.5 \times 10^6 )</td>
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<tr>
<td>10</td>
<td>( 2.6 \times 10^6 )</td>
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</table>
Fig. 3. Settling velocities of minerals in silicate melts. The family of curves 1 through 6 represents the probable range of settling velocities in a granitic liquid having a viscosity of $10^6$ poises and a density, $\rho = 2.3$ g/cc. Curves 4' and 5' contrast settling velocities of plagioclase and pyroxene, respectively, in basaltic liquid ($\eta = 3 \times 10^6$ poises) as given by Hess (1960, fig. 3).

Curve 1. Biotite ($\rho = 3.1$), treated as cylindrical plates with a ratio of diameter (given by ordinate) to thickness of 20:1 and with the shortest dimension oriented in the direction of motion. This ratio is probably extreme, and thus settling velocities would be somewhat greater and could range virtually between curves 1 and 5 depending on axial ratios and/or orientation. Plates of other dimensions and orientations can be treated by the method indicated by McNown and Malaika (1950).

Curve 2. Hornblende ($\rho = 3.3$), treated as cylindrical rods with a ratio of length to diameter of 5:1 oriented with the shortest dimension in the direction of motion. The long dimension is given by the ordinate.

Curve 3. Alkali feldspar ($\rho = 2.6$), treated as spheres.

Curve 4. Plagioclase ($\rho = 2.7$), treated as spheres.

Curve 5. Pyroxene ($\rho = 3.4$), treated as spheres.

Curve 6. Magnetite ($\rho = 5.2$), treated as spheres.

were released, whereas outside this range particles became oriented with their principal cross-sectional area perpendicular to the direction of motion. On the other hand asymmetric particles changed orientation in all ranges of the Reynolds number. It would seem that in the range of Reynolds numbers with which we are concerned for crystal settling in granitic magmas ($R_v < 0.05$) orienting effects would be small, in the absence of convection, unless particular crystals habitually grew with asymmetric distributions of mass, for example, prisms larger at one end.

An incidental point of interest with regard to settling of particles can be made concerning restrictions imposed by the viscosity of the magma on effectiveness of stoping. If one envisions a large magma chamber above which roof rock is dislodged in blocks, say with a nominal radius of 10 meters, at an average rate of 1 block per square kilometer per year the roof would be displaced upward at a rate of about 4 kilometers per million years if the blocks
Fig. 4. Distance travelled versus time for spherical particles with $\Delta \rho = 0.4$ (plagioclase) at $\log_{10} \eta = 6$ and with growth rates $c = \text{d}t/\text{d}t$ (cm/yr) as follows:

\begin{align*}
    c_1 &= 1 \times 10^{-4} \\
    c_2 &= 1 \times 10^{-5} \\
    c_3 &= 1 \times 10^{-6} \\
    c_4 &= 1 \times 10^{-7} \\
    c_5 &= 1 \times 10^{-8}
\end{align*}

The full lines 1 through 7 represent the distances travelled at any given time for each growth rate. The dashed lines represent the loci of points where radii of crystals reach 1 mm and 1 cm, respectively. The dotted lines represent distances travelled by crystals of fixed radii of 1 mm and 1 cm, respectively.

settled rapidly enough. Blocks of this dimension would settle at a rate of nearly 3 kilometers per year in magma with a bulk viscosity of $10^{10}$ poises or far greater distances at even slightly lower viscosities. If stopping ceased before the viscosity of the magma increased greatly, the upper part of the resulting pluton would be relatively free of suspended blocks. These comments do not take into account the effect of the considerable loss of heat to the sinking blocks or the mechanics of fracturing of the roof. Also they imply that the magma has great extension in depth. The problem of heat loss would be more or less offset if convection exists in the magma. The mechanics of fracturing cannot be given a simple general solution. Anderson (1936) has discussed some aspects of the problem with reference to formation of ring dikes and cone sheets, and it
would seem that block stoping is one of the likely consequences of the process envisioned. Mc Birney (1959) discusses a specialized case for fracturing around volcanic necks intruding sandstone. The mechanism is based essentially on the thermal expansion of ground water pore fluid and consequently is not directly applicable to plutonic conditions, although similar effects might be produced due to expansion of intergranular films of water or during possible devolatilization reactions.

*Settling of particles in a suspension.*—Here we have such diverse possibilities that some general relationships can merely be noted. Obviously it is necessary to have some idea of the effect of a given proportion of solid particles on the bulk viscosity of the fluid mixture and how settling rates are affected.

The problem of settling of large crystals in a suspension of small discrete crystals can be approached in a simple way by finding the effective viscosity of the suspension and then by applying the same relationships used in the previous section. The sedimentation of particles in a solid-liquid mixture in which the range of particle sizes is small, however, requires somewhat different considerations.

It was shown by Einstein (1906, 1911) that for small concentrations of rigid spheres the viscosity of a suspension should be given by

\[ \eta_s = (1 + 2.5 \phi) \eta_0 \]

(4)

where \( \eta_s \) is the viscosity of the suspension, \( \eta_0 \) the viscosity of the suspending liquid, and \( \phi \) the volume fraction of the spheres. Roscoe (1952) uses this relationship to predict the viscosity of concentrated suspensions, in which the particles have a wide stepwise size range, by integrating the Einstein relationship over all the sets of particles, each set being a dilute suspension compared with previous sets. The integration gives the simple formula

\[ \eta_r = (1 - \phi)^{-2.5} \]

(5)

where \( \eta_r = \eta_s / \eta_0 \).

Roscoe (1952) was able to modify (5) to predict the viscosity of suspensions of spheres of equal size by utilizing a model proposed by Vand (1948) which in essence takes account of fluid immobilized between spheres in close proximity to one another. The result of the model is to increase the effective volume of the solid, and because of various types of transient groupings, especially at higher concentrations, the overall effect should be predicted by equation (5) if account is taken of the "frozen in" fluid. This is accomplished by means of a simple geometric factor applied to the volume fraction with the result

\[ \eta_r = (1 - 1.35 \phi)^{-2.5} \]

(6)

Equations (5) and (6) are shown by Roscoe to give rather good agreement with the experimental data of Ward and Whitmore (1950) and Eilers (1941) and should provide a useful basis for estimating effective viscosities in magmatic crystal-liquid mixtures. Some values of the viscosity ratio, \( \eta_r \), calculated from (5) and (6) are shown in table 3.

The reason \( \eta_r \) goes to infinity in table 3 is because of the assumption that the particles are perfectly rigid. Thus rigidity develops as the volume fraction approaches unity for the case of widely varying particle size, as a consequence
of the packing characteristics, whereas for uniform spheres rigidity is attained at \( \phi = 0.74 \), corresponding to closest packing of uniform spheres. Of course, the actual behavior of the solid aggregate is not that of a perfectly rigid body. The limiting effective viscosity then would be given by the plastic behavior of the aggregate in question but should be far greater than the fluid viscosities we are considering here.

The effective viscosities in the kind of solid-liquid mixtures we can expect in magmas should lie somewhere between the predictions of equations (5) and (6), depending on the amount of sorting which might be produced by differential settling of crystals. That is, until the magma is more than half crystallized, the bulk viscosity is not increased due to the presence of crystals by more than about one order of magnitude. The effect of shape factors for crystals in granitic magmas is difficult to predict, but insofar as diverse shapes can be considered to be analogous to deviations from a uniform distribution of spheres the resulting effective viscosities should be estimated better from equation (5).

As previously mentioned, the settling of large particles in a suspension is readily obtained on the basis of the modified viscosity and density. In a crystallizing magma, \( \eta_{\text{m}} \) would be a function of time as also would be \( \eta_{\text{o}} \) owing to the changing composition of the liquid. Unfortunately, as stated repeatedly, consideration of several other factors is required in order to carry through a solution for sedimentation during magmatic crystallization, and these additional factors are not yet well understood.

Lewis, Gilliland, and Bauer (1949) have provided an experimentally verified basis for predicting rates of sedimentation of uniform spheres. In this case we are interested in how the presence of other particles affects the settling velocity of an individual particle as it would be predicted by Stokes’ law. We can write their result in the form

\[
v_s = v_o (1 - \phi)^{4.65}
\]

(7),

where \( v_s \) is the sedimentation rate, \( v_o \) is the settling velocity of a particle at infinite dilution, and \( \phi \) is the volume fraction of particles. Some values of \( v_o/v_s \) are given in Table 4. It is seen that the settling velocity predicted by Stokes’ law would be about an order of magnitude high at one-third concentration of even sized crystals and about two orders of magnitude high at two-thirds concentrations of crystals.

Equation (7) is valid at low particle Reynolds numbers (\( \text{Re} < 10 \)) and is adequate for discussion of crystal settling in granitic magmas. It is worth noting, however, that Andersson (1961) gives a comprehensive discussion of sedimentation and presents a useful nomogram of relative settling velocities for conditions beyond the validity of equation (7).

<table>
<thead>
<tr>
<th>Table 3</th>
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<td>The effective viscosity of suspensions of solids, calculated from equations (5) and (6) for particles of diverse size and uniform size, respectively, expressed as the ratio ( \eta_\ell = \eta_\ell^\prime / \eta_\ell^\prime ) for increments of the volume fraction ( \phi ).</td>
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<tr>
<td>( \phi )</td>
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<tr>
<td>( \eta_\ell (5) )</td>
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<td>( \eta_\ell (6) )</td>
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Under magmatic conditions for which a gas phase may exsolve from the melt, it is of interest to know the effect of gas bubbles on the bulk viscosity. It is well-known experimentally that foams are less fluid than the liquid from which they are formed. Sibree (1934) systematized these observations by showing that the equation

$$\eta_r = \frac{1}{1 - (1.2\phi)^{1/3}}$$

accurately reproduces experimental viscosity data to a volume concentration of bubbles ($\phi$) greater than 70 percent. This equation gives results similar to those of equation (5) for suspended solid particles of different sizes. The bubbles in the froths studied by Sibree ranged widely in size so that it appears the effective viscosity of a suspension is surprisingly similar whether the suspended material is in the form of solid particles or gas bubbles.

Murase (1962) concludes on the basis of some theoretical work by Mackenzie (1950) that the presence of bubbles should decrease the effective viscosity. Mackenzie (1950, p. 11), however, noted the possible importance of different boundary conditions and the fact that previous calculations by Taylor (1932) for suspended fluid spheres indicated an increase in effective viscosity. In view of the work by Taylor, Sibree (1934), and others, it appears that extension of Mackenzie’s model for a solid with holes to liquids with bubbles is not valid, and consequently Murase’s (1962) conclusions are suspect. In view of these factors Murase’s data showing lowered viscosities in porous lava are puzzling, and the results may be due to other factors.

It should be noted that the behavior of suspensions may be vastly more complicated than that of Newtonian fluids. Several factors affect the behavior of suspensions in addition to the amount of suspended material. Some of these are size and shape of particles, interfacial properties, and degree of association of the suspended material. In the case of foams it is found that the apparent viscosity varies with the velocity gradient, that is, initially the foam may appear quite rigid, but when it is caused to flow rapidly the apparent viscosity decreases markedly and finally reaches a constant value. It is this limiting value of the viscosity that is represented by equation (8).

Obviously these aspects of the flow of foams must be important during the flow of vesiculating magma, for instance, in volcanic vents. The settling of solid particles through a foam is also complex. In a slowly vesiculating magma it is probable that bubbles would nucleate at solid-liquid interfaces, and consequently crystals might be rafted upward. In a froth, however, the rigidity to low shearing stresses might result in a fairly stable suspension.

### Table 4

Sedimentation rates for suspensions of uniform spheres expressed as the ratio ($v_b/v_o$) of the sedimentation rate to the settling velocity of a single sphere of the same dimension according to Stokes’ law, for increments of the volume fraction ($\phi$) to 0.6 (calculated from equation 7)

<table>
<thead>
<tr>
<th>$\phi$</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_b/v_o$</td>
<td>0.61</td>
<td>0.35</td>
<td>0.19</td>
<td>0.09</td>
<td>0.04</td>
<td>0.01</td>
</tr>
</tbody>
</table>
In contrast with the non-Newtonian behavior of froths, magmatic crystal suspensions should more nearly approximate the properties of a suspension of hard spheres in a Newtonian fluid as long as the crystals are distributed as discrete particles of macroscopic size. Obviously, if the crystals are grouped in clusters the flow properties would be different. It was found by Ward and Whitmore (1950) that the relative viscosity of a suspension of hard, smooth, noninteracting spheres is practically independent of the rate of shear throughout the range of concentrations studied \((0 < \phi < 0.3)\). It is possible that this may not hold true at higher concentrations and consequently the behavior of concentrated suspensions requires more work, especially at volume fractions greater than 0.5. Conclusions concerning the rigidity of crystal-liquid mushes to low shearing stresses must await further experimental work on the rheology of concentrated suspensions.

AN INCIDENTAL COMMENT ON MAGMATIC CRYSTALLIZATION

Mechanisms of crystallization in magmas have received little attention compared with phase equilibria. This is not surprising in view of the fact that the latter are easier to deal with and are of critical importance in an approach to kinetic problems. Crystallization of magmas is, however, largely a kinetic problem within the framework of our knowledge concerning the relative stabilities of phases in magmatic systems. Useful empirical concepts were obtained long ago (for example, see Tammann, 1925) and a number of theoretical analyses of crystal nucleation and growth have been attempted (see Turnbull, 1956; Gilman, 1963). The basic concept frequently cited in geologic literature but infrequently used in any quantitative way is that the rates at which crystal nuclei form and, once formed, grow depend on the degree of undercooling. Stated in a different way crystallization depends on the probability of forming nuclei sufficiently large that surface energies do not cause them to be resorbed. Probabilities are small at low degrees of undercooling because the free energy difference between solid and liquid is small, requiring nuclei of large critical radius. Conversely, at high degrees of undercooling, smaller critical radii are needed because of the large free energy difference, but because of greatly decreased thermal motions the frequency of formation of nuclei is smaller. Thus for some range of either small undercooling or of great undercooling, rates of formation of stable nuclei are small, and between these extremes some maximum rate must exist. Similarly the resultant of the driving force for crystal growth and the motions of the constituents of the liquid produce a maximum rate of growth at some value of the undercooling.

Evidently in order to discuss rates as a function of undercooling we must know the liquidus of a system as a function of pressure and, if the system consists of two or more components, as a function of composition. For example Swift (1947) has found in some cases that linear rates of crystal growth from undercooled silicate liquids are fitted by the empirical formula first proposed by Preston (1940)

\[
\xi = K_1 \frac{(T_L - T)}{\eta}
\]

(9),

where \(\xi\) is the linear growth rate, \(K_1\) is an empirical constant, and \(T_L\) is the
temperature of the liquidus. In terms of the previous comments the undercooling ($T_L - T$) represents the driving force and $\eta$ the resistance to motion. In order to use this equation to discuss growth rates in multicomponent magmatic systems, even assuming its validity in form and a knowledge of $K$, for a given mineral, we would first have to express $T_L$ as a function of composition and pressure and $\eta$ as a function of composition, temperature, and pressure. Neither the equation nor the experimental data are adequate to permit an analytical approach of this sort.

Among the few attempts to interpret crystallization mechanisms in magmas those of Queneau (1902) and Winkler (1949) are particularly notable. Queneau's analysis was based on experience with metallurgical slags combined with a solution of Fourier's heat flow equations for the case of the infinite slab. He concluded, in essence, that for a dike intruded into cold rock, subsequently crystallized minerals should show a continuous increase of grain size inward from the margin. Winkler (1949) took a similar approach but used some data on rates of nucleation and growth and concluded that dikes of certain thicknesses should show a maximum of grain size somewhere between the margin and center for minerals crystallizing in certain ranges of temperature.

In general, it is evident that for crystallization under more or less static conditions the resultant grain size depends on the ratios of growth rates to nucleation rates, $\xi/\zeta$, that existed during cooling. Obviously, larger ratios produce larger crystal sizes and vice versa. Note that if the ratio were constant for some mineral, regardless of the magnitudes of $\xi$ or $\zeta$, the final population of crystal sizes resulting from complete crystallization would be the same regardless of the cooling history.

Some aspects of the problem may be clarified by considering the experimental rate measurements by Winkler (1947) on the crystallization of nephe-

![Graph](image_url)

Fig. 5. Curves for rates of nucleation ($\xi$) and growth ($\zeta$) of nepheline from the melt (full lines) after Winkler (1947). The dashed line is shown to test the effect of a slight change in the shape of the curve for nucleation rate in the region of low undercooling (see fig. 6).
Fig. 6. The ratio of growth rate to nucleation rate expressed as \((\xi/\zeta)^{1/4}\) versus undercooling. The dashed curve shows the effect of the slight change in the curve for the rate of nucleation shown in figure 5.

Winkler obtained a curve for the growth rate (using the distance between prism faces as a measure of size) with a maximum at an undercooling between 20° and 25°C. Similarly he found the maximum nucleation rate between 55° and 60° undercooling (these curves are shown in fig. 5). The form of the curve for the nucleation rate apparently was drawn to correspond to Tammann’s concepts of nucleation. We can use Winkler’s rate curves to obtain the ratio \(\xi/\zeta\), and the results are plotted in figure 6, using the fourth root of the ratio. The ordinate corresponds roughly to the average crystal size on completion of crystallization. Winkler (1947, 1948, 1949) also gives this kind of plot in his papers but shows the curve approaching the origin more steeply in his 1947 paper and apparently carries it to the origin in the 1948 and 1949 papers.

Winkler (1947) also derived a relationship between the cooling rate during crystallization and the crystal size. In his later papers he also shows this curve approaching vanishing crystal size at vanishing cooling rates. This result is strange in that it predicts submicroscopic crystals for cooling rates in the range appropriate for moderately large (kilometer range) intrusive bodies, except for positions near the margins where a sharp maximum of crystal size would exist, assuming no convective mixing. Put more succinctly, the cooling rate-crystal size curves of Winkler suggest the possibility of retaining virtually a glassy state with sufficiently slow cooling. This is indeed a novel idea, but let us examine its foundation more closely.

Winkler’s cooling rate-crystal size curves are an alternate expression of the implications of the ratio \(\xi/\zeta\). That is, for either type of curve the attainment of vanishing crystal size at vanishing undercooling or cooling rate implies that the growth rate, \(\xi\), vanishes more rapidly than the nucleation rate, \(\zeta\). Now, since both \(\xi\) and \(\zeta\) vanish at the liquidus temperature, \(T = T_L\), we have an
indeterminate ratio unless we can demonstrate analytically or experimentally a finite limiting value of $\xi/\zeta$ for $(T_L - T) \to 0$. It is evident, in this regard, that Winkler's (1947) data do not demonstrate the nature of the ratio at small to vanishing undercooling. The data do not extend to sufficiently small undercooling, nor are they sufficiently consistent to permit reliable extrapolation in terms of showing the trend of $\xi/\zeta$. Ratios actually obtained from Winkler’s rate curves suggest an extrapolation of $(\xi/\zeta)^{1/4}$ to a value at about 1 centimeter. If we change the nucleation rate curve slightly in the region of low undercooling we obtain a drastically different result for the trend of the ratio. For example, the slight change shown by the dashed line in figure 5 produces the dashed portion in figure 6 and eliminates the maximum.

It is apparent that conclusions based on ratios of growth to nucleation are extremely sensitive to slight changes in slopes at low undercooling, and conclusions of the kind shown in the solid line in figure 6 must be used cautiously. Winkler’s data do not give us a sufficiently firm foundation for such an interpretation, especially when applied to other minerals, and consequently his interpretation of grain size in dikes is not well founded theoretically or experimentally. It appears to the writer that existing concepts of nucleation suggest that the nucleation rate vanishes more rapidly than does the growth rate, and hence the ratio $\xi/\zeta$ would increase without limit as the temperature $T_L$ is approached. It would be possible, however, to produce a maximum in the ratio $\xi/\zeta$ at greater undercooling, for example, if the maximum growth rate were at a lower temperature than the maximum nucleation rate.

Winkler (1948, 1949) brings to bear some observations on size distributions in dikes which demonstrate maxima near the margins, but the observations would be more convincing if some tests for reliability and significance had been made. It is possible that other factors might produce the same effect, and many more data are needed on size distributions in magmatic bodies and on rates of crystal nucleation and growth for different minerals. In addition, the effect of convective motion on the distribution of crystallizing minerals cannot be discounted, as was done by both Queneau (1902) and Winkler.

We are interested not only in crystal sizes but also in the time required to attain these sizes. Obviously this cannot be worked out rigorously in the light of existing knowledge of $\xi$ and $\zeta$ for minerals of interest. We can, however, estimate orders of magnitude of the time required for crystallization in different parts of a pluton of specified size by the method used by Larsen (1945). This consists of calculating the time required to cool the magma through some interval of temperature in which crystallization took place. Larsen (1945), however, did not take into account the heat of crystallization, and his results are modified somewhat by Jaeger (1957). Rigorously, the solution of the heat flow problem requires a knowledge of crystallization rate rather than the assumption of fixed temperature intervals, for the reasons just discussed. For example, the assumption of a fixed temperature interval obviously does not describe the time required for crystallization at the margin where great undercooling takes place over a short period of time.

For the simple, but unrealistic, case of sudden intrusion into cold rocks of a dike-like pluton 10 kilometers thick at a temperature of 1000°C, crystalliza-
tion at the margin would probably require considerably more than $10^8$ years whereas within a kilometer of the margin there would probably be a minimum time of the order of $10^4$ years which would increase toward the center to the order of $10^8$ years. Because emplacement of such a large mass certainly is not sudden, times would be lengthened in the interior and shortened right at the margin for the reason that earlier in the time of emplacement intruded rocks must have been deeper and hotter.

Convection would alter any patterns of crystallization to a greater or lesser extent by decreasing temperature gradients and by carrying material from one thermal regime to another. Loss of volatiles (mainly $\text{H}_2\text{O}$) from place to place would introduce additional heterogeneities largely in terms of the effects on the liquidus temperatures and the unknown effects on crystallization rates. For example, as has been pointed out by Tuttle and Bowen (1958), crystallization can proceed isothermally as a result of a loss of $\text{H}_2\text{O}$. Under conditions in which loss of $\text{H}_2\text{O}$ is important the rate of crystallization then depends on the rate of diffusion of $\text{H}_2\text{O}$ as well as of heat. Data are lacking at present, however, to permit an evaluation of the rates of diffusion of $\text{H}_2\text{O}$ in magmas. Ingestion of $\text{H}_2\text{O}$ would lower the liquidus locally and would change crystallization rates and the temperature interval and conceivably, depending on diffusion rates, could locally reverse the expected normal trend of progressive crystallization from near the margin inward.

**Convection**

*Forced convection in channels.*—Boundary conditions for flow of magmas are also widely variable and poorly known. Again, however, it may be worthwhile to examine results for simple cases. One which gives some insight into magmatic mobility is flow in channels in response to a pressure gradient. Forced convection simply means mass transfer of fluid as a result of an externally applied force. Natural convection, on the other hand, results from the action of a gravitational field on a fluid in which the density is not constant, as is the case when a chemically homogeneous fluid is subjected to a temperature gradient.

The simplest case of forced convection is the incompressible single-phase flow of a viscous fluid in a cylindrical conduit under a constant pressure gradient. Under these conditions the quantity of flow is given by Poiseuille's law

$$Q = \frac{\pi P a^4}{8 \eta l} \quad (10),$$

where $Q$ is the volume rate of flow, $a$ is the radius of the cylinder, and $l$ is the length. Figure 7 shows solutions of this equation for an arbitrary length of 10 kilometers and a pressure gradient of 1000 bars. Obviously for these simple boundary conditions, large volumes could be transported in conduits of geologically modest dimensions in geologically short times under conditions of strong forced convection.

Equation (10) is valid only for streamline flow which, however, must become unstable with respect to turbulent flow under some conditions. These conditions generally are defined with reference to experimental data and are
described by the familiar dimensionless Reynolds number \( R_e = 2 \frac{u v}{\nu} \) where \( v \) is the average velocity of flow given by \( Q/\pi a^2 \) so that, restated in terms of the average volume rate of flow,

\[
R_e = \frac{2 \rho Q}{\pi a \nu}
\]

(11).

Data giving the critical Reynolds number for which streamline flow in straight smooth cylindrical conduits becomes unstable with respect to turbulent flow have been summarized by McAdams (1954, table 6-2), and for our purposes the critical value is about \( 2 \times 10^5 \). On the basis of (11) we can calculate ratios of volume flow to diameter which give \( R_e = 2 \times 10^5 \). Such ratios are plotted as a dashed line in figure 7, and the intersection with the flow curve gives the upper limit for stable streamline flow.

In the case of magma with suspended crystals \( \nu \) would be the effective viscosity of the suspension and \( \rho \) its mean density. Obviously, then, turbulence in flowing magma would be sensitive to the distribution of suspended crystals.

The above relationships must be modified for flow in conduits of rectangular cross sections, that is, appropriate for conclusions concerning flow in dikes. The relationship for volume rate of flow now becomes
where \( s_1 \) and \( s_2 \) are the sides of the rectangular cross section, \( l \) is the distance normal to the cross section, and \( \Psi \) is a function of the ratio of the sides. Values of the function \( \Psi \) are given in graphical form by McAdams (1954, fig. 6-5), and for conduits with very small ratios of \( s_1/s_2 \) \((<0.1)\) can be represented by

\[
\Psi = 0.32 \frac{s_1}{s_2} \simeq \frac{s_1}{3s_2} \tag{13}
\]

Results for a dike-like conduit are also shown in figure 7 where the conduit is tabular with two dimensions of 10 kilometers and the third dimension given by the ordinate. The limiting conditions for streamline flow for this case are given by

\[
R_e = \frac{4r_h \rho v}{\eta} \tag{14a},
\]

or for volume rate of flow

\[
R_v = \frac{4r_h \rho Q}{s_1 s_2 \eta} = \frac{2 \rho Q}{(s_1 + s_2) \eta} \tag{14b},
\]

where \( r_h \) is the so-called hydraulic radius given by \( r_h = s_1 s_2/(2s_1 + 2s_2) \). The limiting volume rate of flow for streamline flow in the tabular conduit is also shown in figure 7 as the point of intersection between the curve for volume rate of flow versus \( s_1 = D \) (for \( R_e = 2 \times 10^3 \)) calculated from equation (14b) and the curve for flow rates calculated from equation (12).

Adjustments of curves in figure 7 for other conditions and dimensions are made readily by reference to equations (10) through (14). The quantity of flow is inversely proportional either to viscosity or length (that is, an order of magnitude increase in either decreases \( Q \) an order of magnitude) and is directly proportional to the pressure gradient.

An application of flow rates that may have some geologic significance is to estimate how rapidly dikes can form under certain conditions. Given a tabular fissure, we can estimate the elapsed time for formation of a dike if we apply the artificial assumption that the fissure opened to the magma instantaneously and subsequently did not change dimensions. The volume rate of flow into the fissure is \( Q = dV/dt \), where \( V \) is the volume of the fluid in the fissure at time \( t \). Assuming a constant pressure gradient we can treat small increments during the process of filling by means of equation (12). Thus \( Q \) can be expressed as a function of the volume occupied by fluid, and the elapsed time is obtained from the integral

\[
t = \int_0^V \frac{1}{Q} \, dV \tag{15}.
\]

Results are shown in figure 8 for dikes of various thicknesses.

As usual there are several problems in applying results for idealized cases. The most difficult to evaluate are the conditions of stress and strain that give rise to intrusive bodies of a particular geometric type. In the case of dikes, instantaneously opened fissures are unrealistic, and the actual process probably
Fig. 8. Time required to form dikes of various thicknesses ($s_1$) under conditions of instantaneous fracture and opening of a fissure, for $\eta = 10^5$ poises, $\Delta P = 1000$ bars, and $s_2 = 10$ km.

involves rapid propagation of a fracture which is then wedged open by magma depending on the pressure gradients established. Thus the rapidity with which a dike can form would normally be less than that indicated in figure 8 by some factor.

Other problems arise with regard to roughness of walls and curvature of conduits. Rough walls facilitate transition to turbulent flow at lower Reynolds numbers. Pits and protuberances along walls of conduits have little effect on the results, however, if they are only of the order of 1 percent of the diameter or thickness. Curvature, on the other hand, tends to increase the critical Reynolds number according to data given by McAdams (1954, table 6-3). Another important factor in forced flow in conduits is the nature of the flow at the orifice. In general we might expect large disturbances in this region which might be damped out farther along the conduit, depending on the usual variables previously discussed. For example, equation (15) does not account for the fact that the first increments entering the fissure would have high velocities and the initial flow would be turbulent. Turbulence would be damped out within a short distance in narrow fissures, and the results probably would differ little from those shown in figure 8. Predictions of laminar versus turbulent flow during the emplacement of thick dikes and necks, however, would have to take this into account.

Thus it seems possible that under certain geologic conditions forced turbulent flow of salic magma could occur. It is difficult, however, to imagine how evidence of turbulent flow of magma might be preserved in the resulting crystalline rock (compare discussion by Spry, 1953). If the magma were vir-
tually entirely liquid, turbulence would merely produce better mixing which would result in a homogeneous rock. As flow slows due to the effect of decreasing temperature on the viscosity or in response to the mechanical behavior of the conduit system, turbulence would be damped out, and normally the final stages of motion would be expected to be laminar. In either case evidence in the rock bearing on the kind of motion depends on markers of some sort, for example, uneven distribution of crystals present during flow, sharp compositional gradients in the liquid magma (certainly not expected to persist long if flow were turbulent), or the presence of prismatic or other crystals capable of showing preferred orientation. Evidence in the form of preferred orientations of crystals, however, requires experimental demonstration of the orienting effects of laminar and turbulent flow. It appears to the writer that preservation of evidence of turbulent flow requires an exceedingly transient condition, for example, magma bursting into an open cavity where it is so quickly chilled that the large entry disturbance had insufficient time to be smoothed out.

It is interesting that, with reference to figure 8, formation of even thin dikes could take place with great suddenness. For dikes that are intruded quickly into cool rocks crystallization would not be a factor because we would be dealing with a supercooled liquid and only the temperature coefficient of the viscosity would have to be introduced.

The flow properties of solid-liquid mixtures or of froths would be estimated by adjusting the bulk viscosity as previously indicated. There would, however, be a tendency for solid particles near the walls to migrate toward the central portion of the conduit as indicated by the theoretical analysis given by Jeffrey (1922), Starkey (1956), and others. It is interesting that careful studies by Segré and Silberberg (1962) and Karmish, Goldsmith, and Mason (1963) predict the development of a maximum in the concentration of particles roughly midway between the axis and the walls of a tubular conduit. All these effects, however, would be negligible for viscosities and flow velocities considered here. Under certain conditions of flow of magma of much lower viscosities, however, these factors should become important (cf., Bhattacharji, 1963). It is evident that the radial distribution of particles depends on the size and number of particles as well as other factors, and the conclusion that particles move toward the axis of a conduit is an oversimplification. A completely general analysis of the problem is not available but the references cited, together with an additional comment by Starkey (1962), give considerable insight and, perhaps with some additional experiments, should permit conclusions of considerable geologic interest.

Perhaps one of the more geologically important aspects of the flow of suspensions relates to the orientations of particles. This problem has been treated theoretically by Jeffrey (1922) and experimentally by Goldsmith and Mason (1962). The experimental results for suspensions of rigid rods showed that most particles became oriented with their long axes subparallel to the direction of stream line flow. Presumably something similar would hold for plates, that is, the plane of the major axes would tend to parallel the stream lines. Additional experimental work concerning particle orientations, however, would be useful.
Natural convection.—A case of natural convection of considerable geologic interest, which can be tested approximately, is the possible gravitative flow of material in magma having the form of a cylindrical stock or neck connected to a large magmatic reservoir at depth. In such a system the temperature gradient would produce a density gradient and consequently a tendency for a downward motion of fluid near the walls and an upward motion of fluid nearer the center. Properties of flow and heat transfer appropriate to this kind of fluid system have been worked out theoretically by Lighthill (1953), and the theoretical predictions have been verified to a remarkable degree by the experiments of Martin and Cohen (1954). An important geologic application of Lighthill’s theories was made previously by Mc Birney (1959) with respect to emplacement of basic volcanic necks, and the present comments are similar but are directed toward more highly viscous magmas. The purpose, of course, is to try to discover if natural convection can exist in bodies of salic magmas for any geologically reasonable dimensions, viscosities, and temperature gradients.

Lighthill’s (1953) results are given in terms of relationships involving various dimensionless ratios. Before applying his results, however, some discussion is in order concerning the use of dimensionless ratios in fluid mechanics. In general, a quantitative description of the transfer of both heat and matter is a function of many variables. It is possible, however, with complicated functions to group the variables and consider only the functional relationships between these groups. This may obscure the physical significance of such groups but makes the mathematical description of a complicated process, such as convective heat transfer, tractable. Once a simple relationship between groups is found it is readily seen how changes in the specific physical variables will affect the result. Further insight into the techniques of dimensional analysis can be obtained from many sources, but see especially Klinkenberg and Moooy (1948) and Bosworth (1952).

Among the many possible dimensionless ratios in fluid mechanics certain ones are standard and are given names after prominent workers in the field (for example, the Reynolds number previously cited). Here we are interested in the interdependence between only a few of these numbers and what this can tell us concerning the possibilities of convection and the magnitude of heat transfer in a convecting system. Illustration of Lighthill’s (1953) results are shown in terms of the following dimensionless ratios:

the Nusselt number based on tube radius, a.

\[ N_a = \frac{Q_o}{\pi k (T_o - T_1)} \]  (16);

the modified Grashof number based on tube radius

\[ G_a = \frac{\alpha (T_o - T_1) ga^3}{\nu k} \]  (17);

the Prandtl number

\[ \sigma = \frac{\nu}{\kappa} = \frac{\gamma C_p}{k} \]  (18);

and the non-dimensional temperature

\[ T_n = \frac{ag a^4 (T_o - T)}{\nu k l} \), (laminar flow) \]  (19).
In (16) $Q_b$ is the total rate of heat transfer from the tube, and $(T_o - T_1)$ is the temperature at the center of the tube less the temperature at the margin (this is the reverse of Lighthill's convention, since we are considering a cooled rather than a heated fluid). The other symbols are conventional quantities and are given in table 1. For the purposes of our approximate calculations the following values are used and are only estimates on the basis of data given by Birch, Schairer, and Spicer (1942):

$$\alpha \simeq 5 \times 10^{-5} \text{ deg}^{-1}$$
$$C_p \simeq 0.26 \text{ cal gm}^{-1}\text{deg}^{-1}$$
$$k \simeq 5 \times 10^{-8} \text{ cal cm}^{-1}\text{sec}^{-1}\text{deg}^{-1}$$
$$\kappa \simeq 8 \times 10^{-3} \text{ cm}^{2}\text{sec}^{-1}.$$  

The Prandtl number is large because of the high viscosity and low thermal conductivity, and negligible error is introduced by considering it infinite. On this basis we can examine some of Lighthill's results which provide a basis for considering the type of flow and for evaluating $N_a$, and thus from equation (16), the total heat transfer rate. Lighthill's results are summarized in figures 9 and 10. Figure 9 gives the relationship between $N_a$ and $T_n$ for laminar flow.

![Diagram](image_url)

**Fig. 9.** Theoretical regimes of laminar natural convection in vertical tubes closed at one end and open into a reservoir of fluid at the other, expressed in terms of the Nusselt number ($N_a$) and non-dimensional temperature ($T_n$), after Lighthill (1953): a, flow with boundary layer not filling the tube; b, region of transition between types a and c; c, flow with boundary layer filling the tube; d, flow with boundary layer filling the tube except for a stagnant portion at the closed end.
Fig. 10. Theoretical regimes for natural convection in vertical tubes for laminar (region L) and turbulent (region T) flow assuming large entry disturbances, expressed in terms of the Nusselt number (Na) and modified Grashof number (Ga), after Lighthill (1953). The solid and dashed lines give results for ratios of length to tube radius of 5, 10, 20, 50, 100, and 200. The limiting solid curve in region T is for the regime in which the boundary layer does not fill the tube. The dotted lines represent transitions from laminar to turbulent flow (on left) and from flow filling the tube to flow not filling the tube (on right). A regime of flow with a stagnant portion would occur in the lower left quadrant of the diagram, even in the region of turbulent flow at ratios of length to radius of about 50 and greater.

only, whereas figure 10 shows both laminar and turbulent regions with respect to Na and Ga for the reasonable assumption that large disturbances exist where fluid enters the tube. Three regimes of flow are recognized in figure 9, which from the bottom upward correspond to conditions approaching stagnation, slow convection throughout the tube, and rapid convection near the walls. The transition region implies a discontinuous change from the second to the third regime, and this was observed experimentally by Martin and Cohen (1954) in terms of oscillations between the two types of flow. The reentrant portions of the curves in figure 10 are a result of transition from laminar to turbulent flow in the regime where the convection cell fills the whole tube. In these

| Table 5 |
| Values of the modified Grashof number and non-dimensional temperature for log₁₀η = 10, l = 1 × 10⁵ cm (1 km), and (T₀ − T₁) = 300°C.* |
| (1) log₁₀(l/a) (cm) | 2 | 3 | 4 | 5 | 6 |
| (2) log₁₀(Ga) | -0.3 | 2.7 | 5.7 | 8.7 | 11.7 |
| (3) log₁₀(T₀) | -3.3 | 0.7 | 4.7 | 8.7 | 12.7 |
| (4) l/a | 10⁰ | 10⁴ | 10 | 1 | 0.1 |

* For other viscosities algebraically add one positive log unit to each entry in rows (2) and (3) for each order lower than 10 and one negative log unit for each order greater than 10.

For other temperature gradients algebraically add log [(T₀−T₁)/300] to each entry in rows (2) and (3); that is, for (T₀−T₁) = 100 add -0.48 to each entry; for (T₀−T₁) = 1 add -2.48, et cetera.

For other lengths, 1 (cm), algebraically add (5 - log l) to log T₀; log Ga will be unchanged. The ratio l/a, of course, is multiplied by the factor 1 × 10⁻⁴ l.
regions laminar flow gives the more efficient heat transfer whereas the reverse is true when both laminar and turbulent flow are of the type where the boundary layer does not fill the tube.

Table 5 gives some values of the modified Grashof number and nondimensional temperature for a constant viscosity of $1 \times 10^{10}$ poises and length of 1 kilometer for various tube radii. Comparison of these numbers with figures 9 and 10 is instructive. For a 1 meter radius the fluid is probably stagnant, and for a 10 meter radius, at least partly stagnant in the upper part. Between 10 meters and 100 meters, however, we pass into the laminar boundary layer flow or turbulent boundary layer flow, depending on the entry disturbance. At yet greater radii turbulent boundary layer flow would be expected.

The conclusions of the preceding paragraph require the viscosity and the temperature gradient to be constant. Obviously this is artificial because the viscosity is temperature dependent and also because the central and boundary temperatures within magmatic bodies are not artificially controlled. The fact that viscosity is not constant would alter the velocity profiles, and consequently the relationship between tube radii and the regime of flow is only an approximation. A rather high average viscosity was chosen in table 5 so that the estimated ranges of tube radii for a given regime should not have to be revised upward very much for the probable range of viscosities of granitic magmas. Actually in the case of magmas with high H$_2$O content revision to smaller radii might be required.

The convection system established for any given temperature gradient is necessarily a transient condition unless convective heat transfer is exactly balanced by conductive heat loss through the walls. If heat transfer by convection exceeds heat loss by conduction through the walls the marginal material is heated and the temperature gradient decreased so that eventually convection will die out. On the other hand if conductive heat transfer through the walls exceeds convective heat transfer, the magma is cooled with the result that viscosity is increased, crystallization begins, and eventually the temperature gradient is decreased.

These comments can be put on a more quantitative basis by graphically comparing heat transfer by convection and conduction, respectively, in a manner similar to that given by Mc Birney (1959). The heat necessary to maintain a constant wall temperature can be estimated from the equation given by Ingersoll, Zobell, and Ingersoll (1948, equation 9.8d) for the line source of heat

$$T = q \frac{1}{2\pi kr} \int_0^\infty \frac{e^{-\beta^2}}{\beta} d\beta = \frac{q}{2\pi k} I\left(\frac{r}{2\sqrt{kt}}\right)$$

(20),

where $r$ is the radial distance from the line, $t$ the time, and where the integral $I\left(\frac{r}{2\sqrt{kt}}\right)$ is tabulated (Ingersoll, Zobell, and Ingersoll, 1948, app. F).\(^1\)

\(^1\) It appears that Mc Birney (1959, equation 2) inadvertently gave the equation for the point source of heat rather than the line source.
Figure 11 shows solutions of equation (20) for various times and tube radii required to give a constant boundary temperature of 500°C (full curves), together with a solution for total convective heat transfer rate solved using figure 10 and equation (16) (broken curve). The actual values shown are not to be taken strictly, as there are numerous complications. The graph shows, however, that above the dashed line conductive heat loss is greater than convective transfer and consequently a strong temperature gradient would be maintained which would promote continued convection until the viscosity of the magma was greatly increased due to cooling and crystallization. Below the solid line convection is more effective than conduction, and consequently the wall rocks would tend to be heated up.

Note that regardless of tube radius, conduction is initially more effective than convection. At a radius of about 100 meters convective heat transfer tends to catch up with conduction after a short time, and this might result in increased temperatures at the margin if convection lasted long enough. For tube radii greater than about 200 meters conduction would be the more effective for long periods of time. Obviously, during the time that conduction overbalances convection the properties of the magma must be changing as a result of cooling, and these changes would tend to suppress convection. Thus the dashed curve would indicate too great a heat transfer, and consequently it is problematical whether convective heat transfer would ever equal conductive heat loss in tubes of any radius. In order to answer this question we would have to obtain a general solution for temperature distributions in a convecting system.

![Graph](image)

**Fig. 11.** Comparison of heat loss by conduction (solid lines) and heat transfer by natural convection (dashed line) in a tube with a length (l) of 1 km and radius given by the abscissa. Heat loss was computed (from equation 20) so that the temperature at the boundary remains constant at 500°C. Heat transfer by convection was computed from figure 10 and equation (16).
and include variables describing the changes in the viscosity as a result of cooling and the development of crystals. This task is too formidable to attempt here. We can say, however, that for the larger diameters natural convection may proceed for a long time in view of the probable rates of crystallization, perhaps throughout much of the time required for crystallization. Since crystallization probably proceeds near margins the density gradient is enhanced with consequent impetus to convection (compare, two-phase convection of Grout, 1918). Note also that in magmas initially undersaturated in H₂O crystallization leads to enrichment of the liquid in H₂O which tends to counteract increasing viscosities at the margins due to cooling and the presence of crystals (compare, fig. 1).

The previous remarks assume a large underlying reservoir of fluid. Natural convection is also possible for cases in which no reservoir exists, but it would be damped out much more quickly. Also in actual cases we would expect more complex convection cells than envisioned above. For example crystallization near the roof of a magma chamber of large diameter might produce downward streaming of crystal-liquid mixture in places, but this is difficult to predict quantitatively. Convection in dike-like bodies would be similar to convection in tubes, but, depending on disturbances of one sort or another laterally along the dike, would probably consist of numerous cells.

One consequence of convection with an underlying magma reservoir is that greater heat flow is obtained in the rock around the neck or dike. Thus, calculation of cooling of such igneous bodies as well as temperatures in the vicinity of the intrusion must take this into account or contain errors of varying magnitudes. The case of natural convection in salic magmas is also of interest with regard to temperatures and duration of activity in hot springs areas, for example, Yellowstone. Wairakei. Donald E. White of the U. S. Geological Survey (oral communication, March, 1963) first called the writer’s attention to possible discrepancies between heat transfer by hot springs and heat flow due to static cooling of underlying magma, and he suggested magmatic convection as a possible explanation. Banwell (1963) also considers convection in magmas in connection with predictions of available energy for geothermal power. In view of the preceding discussion of natural convection, the speculations by White and by Banwell appear to be real possibilities even though it is difficult to predict accurately the duration of convection and the actual heat transfer.

McBirney (1959) concluded that natural convection in tubular conduits could, under appropriate conditions of stress in wall rock suitable for fragmentation, provide an effective mechanism for emplacement of volcanic necks. The process amounts to a natural inverse drilling mechanism and, in the opinion of the writer, is an important concept for some kinds of magmatic intrusion. Something similar might operate under some deep-seated conditions for salic magmas, but evaluation of its possible effectiveness is even more difficult than for emplacement of shallow volcanic necks.

**SPECULATIONS ON THE FLUID MECHANICAL HISTORY OF SALIC MAGMAS**

Several comments have been made in the preceding discussions on conclusions that appear to be geologically applicable. It is no doubt obvious at
this point that geologic boundary conditions are usually so vague that specific applications are difficult. An important example of the vagueness of boundary conditions with which we are faced is the general uncertainty of H₂O content of magmas. We have a fair knowledge of saturation limits for H₂O in silicate melts under widely varying conditions but little knowledge of the degree of undersaturation with regard to H₂O in magmas. Thus, viscosities chosen for discussion in this paper are based only on a range of possibilities indicated by figure 1, and in any given case the choice was arbitrary. Presumably residual liquids approach or reach saturation, and consequently viscosities then follow a path similar to that shown by the dashed line in figure 1. Vapor pressures of volatile components of mineral assemblages offer some promise for demonstrating activities of H₂O in magmas, and consequently this gap may be filled so that we can predict viscosities more confidently.

The conclusions reached early in the paper concerning settling rates assumed more or less static conditions. Somewhat different results would be expected under conditions of convection. These results would depend on the types and duration of various regimes of flow. Several of the possibilities for regimes of flow are shown schematically in figure 12. Types 1 and 2 are possibilities for laminar flow regimes where the amount of stagnation at the top would probably depend on vertical temperature gradients. Type 3 depicts vortices near the top which may develop to a greater or lesser extent in any of the regimes of flow. Type 4 depicts the turbulent boundary layer regimes.

Crystals which grow along the margins will either be attached to the walls (consequently decreasing the radius and eventually choking off convection) or be carried into the reservoir. If the temperature gradient is strong, crystallization will be much slower at the margin than some distance in and attachment would seem unlikely. Crystallization at the top could produce several results. For convection of type 1 normal crystal settling would be approximated in the

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Fig. 12. Schematic diagrams of types of natural convection in tubes connected with an underlying reservoir. The type depends on conditions as outlined in figures 9 and 10.

Type 1. Laminar boundary layer not completely filling tube.
Type 2. Stagnant portion above laminar boundary layer.
Type 3. Laminar boundary layer filling tube, with ring vortices at top.
Type 4. Turbulent boundary layer almost completely filling tube.
center while, in the regions of overturn, crystals would be carried down along the walls. Near the roof above the convection cells great fractionation could occur because early formed crystals would be caught up and carried away after settling only short distances. If progressive stagnation at the top occurred, zones of differentiation would be formed in these regions, depending on the duration of convection. If the convection cells virtually filled the chamber this differentiation could proceed more or less uniformly from the top. If the cells were developed mainly near the margins, flow velocities would be rapid, and, if early formed crystals could adhere near the walls, differentiation by the diffusion mechanism suggested by Hess (1960) could conceivably operate, also for the reason that short distances would be required for material transfer.

Rates of flow differ depending on the dimensions of the cells. A rough idea is given by considering mass transfer required for a given heat transfer. For a temperature gradient of $300^\circ C$ and heat transfer rate of $10^8$ cal/sec about $5 \times 10^5$ cm$^3$/sec must be cooled. If the radius is 1 kilometer and length 1 kilometer and the upward and downward flow is about equally divided, the average velocity up or down would be about 10 m/yr. Reference to figure 3 shows that this velocity is probably higher than settling velocities for crystals of reasonable size especially if the viscosity is higher than $10^6$ poises. Since this is an average velocity the motion of crystals in the center might have either a slight upward or downward component while crystals in intermediate position would move upward and crystals near the side would have a very large downward component. For smaller cells actual velocities in the main regions of flow could be much greater than 10 m/yr.

The possibility of vortices of type 3 in figure 12 is interesting. The overturn in these vortices might be quite rapid, depending on the temperature (thus viscosity) gradient. If the vortices persisted during the early stages of crystallization strong fractionation might take place leaving zones with strong composition gradients. Whether or not these could develop and persist would depend on the balance between settling (and convective flow) producing fractionation and diffusion tending to minimize the composition gradient.

In regard to vortices of type 3 a somewhat novel speculation suggests itself. Consider basic magma in convective circulation. Given the proper dimensions and temperature gradient near the roof, pockets of strongly fractionated liquid, for example, perhaps pockets of salic liquid in basalt, might develop. The writer has no convictions in this regard, but it is evident that the mechanics of motion in magmatic systems frequently can be expected to produce seemingly anomalous associations such as the rhyolite-basalt association.

The main effect of turbulence, as indicated in type 4 of figure 12, in addition to carrying material downward near the walls, is a general stirring in the central portion of the magma. This might result in differentiation progressing downward from the roof, but with a fairly uniform composition in horizontal section.

Layering, of course, would not occur in these kinds of convection, except, perhaps, locally on a small scale, because of the reservoir. For magma chambers without connections to reservoirs we might expect evidence of layering at the bottoms, but, for salic magma, probably only large plutons would permit
natural convection and generally only small portions of these are available for study, with the result that vertical distributions of composition are not well known.

Crystallization in the absence of convection would produce a pattern of differentiation inward and downward by settling with more basic compositions near the margins and at depth. In this case the greatest differentiation would be found in the core where crystal growth was slow and where accumulation of H₂O and other volatiles would tend to occur.

CONCLUSION

In a sense, the preceding commentary may be construed as a catalogue of deficiencies in our knowledge of magmatic systems. These are perhaps about equally divided between insufficient quantitative data from the field and an inadequate foundation in theory and experiment for their interpretation. Many of the latter have been pointed out already, but it is the problem of quantitative measurements from the field that is perhaps the major obstacle in view of the difficulty and time required to carry out measurements in sufficient detail. The mechanistic approach to problems of magmatic evolution may seem to offer impossible goals, and consequently gathering data toward these ends may seem fruitless. Consider, however, that it is precisely the elucidation of the mechanical process by which an igneous rock is produced that is one of the chief aims, usually only by implication, of petrogenetic studies. The work that is needed most is to extend observations of a kind already made as a routine part of petrologic studies, that is, formulate on a more quantitative basis the description of an igneous rock body with regard to all chemical, mineralogical, and textural variations within a quantitatively described geologic setting. These results can provide us with several guides to an interpretation of the mechanical history of the rock body. The proportions of the major minerals together with their compositions (taking into account the proportions and compositions in zoned minerals) give a basis for inferring the path of crystallization, temperatures, and H₂O content of the melt in the light of experimental data on solid-liquid equilibria and equilibria between minerals as a function of vapor pressures of O₂ and H₂O. Thus we can conceivably describe the changes in viscosity as a function of temperature, melt composition, and proportions of crystals. The quantitative distribution of minerals can give us a basis against which to compare concepts of crystallization and differential movement between crystals and liquid. Size distributions are needed in both treatment of crystallization rate and crystal settling velocities. Data on the geometry of the igneous body and its extension in depth are needed to consider mass flow of magma by forced or natural convection and the related problems of heat transfer.

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