PRELIMINARY NUMERICAL ANALYSIS OF
PROCESSES RELATED TO MAGMA
CRYSTALLIZATION AND STRESS EVOLUTION IN
COOLING PLUTON ENVIRONMENTS

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ABSTRACT. Post emplacement stress evolution and magma crystallization history have been simulated for a single pluton-host rock system. These processes were analyzed by equations representing tectonic, gravitational, magmatic, and thermal processes and their temporal interactions. Then these partial differential equations were solved using the finite-domain element method. To determine the abundance and orientation of fractures as a function of time and space, stress values were then compared with failure criteria.

The calculations indicate that large increases in magma pressure occur as regions of crystals + liquid + vapor form within the chamber. The magnitude of this pressure increase is a function of both the water content and thermodynamic state of the magma. In the system considered, large pressure increases occur when approx 8 volume percent of the system develops a three-phase region. As this development continues, the rocks adjacent to the magma are stressed to the point of tensile failure. These fractures are predominantly vertical at the top 1 km of the pluton but are nearly horizontal at greater depths. Pressure release accompanying fracturing would be evident as rapid crystallization textures in the pluton. Multiple magma chambers form in the numerical model from a single chamber 17,000 yrs after emplacement. These effects should be evident in plutons as textural variations that define composite map units.

Magma pressure and differential thermal expansion are predicted to cause a radial-tangential pattern of stress centered around regions within the pluton that have experienced the greatest temperature decrease. Although thermal stresses cause tensile and shear fractures within the plutons they cause only shear fractures within the host rocks around the top of the pluton. Fractures produced by thermal and magma pressure processes have similar orientations within the pluton. Both these fracturing processes occur simultaneously at the top of the pluton, but with depth, fractures caused by thermal processes lag increasingly farther behind fractures caused by increasing magma pressure, until there is a 200,000 yr time lag between the two failure conditions at the bottom of the pluton. Significant temporal and spatial variations in dike orientations and fracture controlled permeability and porosity seem to be an inevitable consequence of thermal energy dispersion from magmas. These processes will augment the porosity increases caused by differential pore-fluid expansion previously discussed by Knapp and Knight (1977).

INTRODUCTION

The thermal, mechanical, and chemical consequences of magma emplacement, crystallization, and cooling in the crust are fundamental to understanding ore deposition, metamorphism, rock deformation, and tectonics. The evolution of stress in these environments determines the abundance, orientation, and effective apertures of fractures. These fracture characteristics determine the magnitude and homogeneity of rock permeability and consequently control the style of transport processes. Because of the fundamental importance of stress and strain in these systems, equations that describe thermal and mechanical processes subsequent to intrusion are derived and solved by finite element methods. Then a hydrous magma-host rock system that cools by pure conduction is analyzed in order to predict how stress and magma crystallization might evolve in the natural analog to this system.

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Stress evolution and formation of fractures in cooling pluton environments have long been recognized as major controls in the formation of ore deposits (Ransome, 1919; Lindgren, 1933; Newhouse, 1942; McKinstry, 1955; and Wisser, 1960) and in granite tectonics (Balk, 1937). Systematic fracture distributions are commonly observed in the host rocks around plutonic bodies (Anderson, 1936), and systematic orientations of fractures and dikes are common within and around plutons in the Basin and Range Province of Arizona (Rehrig and Heidrick, 1972, 1976). Variations in the orientation of fractures with time and temperature are recorded by cross-cutting vein mineral assemblages and their included fluids (Tittley, Fleming, and Neale, 1978; Tittley, 1975; Haynes and Titley, 1980). The orientation, abundance, and continuity of these fractures suggest that there was a dramatic variation in the magnitude and orientation of stress during the post-intrusion thermal history of plutons.

The state of the magma during the post-intrusion cooling period has been considered previously with respect to the large volume changes associated with the dissolution of H₂O from the melt (Burnham, 1967, 1979; Putnam and Alfors, 1969; Czamanske and Wones, 1973; and Whitney, 1975, 1977). These studies suggest that the state-of-stress in these environments is a direct consequence of the dispersion of thermal energy away from the magma.

Inflation of a magma chamber during magma emplacement also causes deformation and fracture of the host rocks, stoping of roof and wall rocks, and formation of sills and dikes (Shaw, 1979; and Petraske, Hodge, and Shaw, 1978). Fracture orientations produced in host rocks by a static and arbitrarily pressurized magma have been analyzed using equations that describe equilibrium stresses in an elastic, two-dimensional plate containing a constantly pressurized hole (Anderson, 1951; Odé, 1957; Robson and Barr, 1964; Cummings, 1968; Roberts, 1970), and the analogous problem for a three-dimensional domain has been analyzed by Koide and Bhattacharji (1975). However, these studies do not account for the time variation in stress conditions, nor temporal effects of magma pressure, nor thermal stresses. Other than the conceptual contributions of Hulin (1948), Mc Birney (1959), Spry (1962), Lister (1974), and Voight and others (1978), a quantitative model that simulates the stress history observed within plutons is lacking.

Recent advances in the solution of heat and mass transfer problems (Norton and Knight, 1977) and structural analyses by the finite-domain element method (Zienkiewicz, 1971) permit numerical simulation of thermo-mechanical processes and simultaneous changes in the thermodynamic states of magmas in crustal environments. It is through the pioneering efforts of Bowen (1928), Goranson (1931), Tuttle and Bowen (1958), and most recently Burnham (1967, 1979), Burnham and Davis (1969, 1971, 1974), Piwinski and Wyllie (1970), and Robertson and Wyllie (1971) to obtain equation of state data on magmas that the importance of magmatic processes in hydrothermal systems can now be studied. Most
of the magmatic related mechanical processes investigated in this study were first recognized by Burnham and his coworkers.

CONCEPTUAL MODEL

The evolution of stress and strain in hydrothermal systems is evidently caused by tectonic, gravitational, magmatic, and thermal forces. Tectonic forces generate the regional stress field into which magmas intrude and control the shape of the magma chamber. Gravitational forces arise from density contrasts between the hot, relatively low density pluton and the cooler, relatively high density host rocks. These body forces are largest during inflation of the chamber and act with diminishing importance as the pluton becomes stationary and cools in situ. Magmatic forces are caused by the changes in specific volumes of the magma during cooling and crystallization; these volume changes cause expansion or contraction forces on the surrounding rocks and large changes in the stress field. Because plutons are only partially molten for a small fraction, approx 10 percent, of the active history of a hydrothermal system (Norton and Knight, 1977; Norton and Taylor, 1979), the magmatic forces are of relatively short duration. However, they have a large effect on the evolution and form of the igneous rocks during this small time interval and subsequently on the hydrothermal system. Thermal forces originate from temperature contrasts between and within the hot pluton and cooler host rocks; heat flows from the cooling pluton into the host rocks and causes contraction of the pluton and relative expansion of the host rocks. Therefore, tensile stresses tend to develop at the boundary between these environments.

These contributions to stress and strain are closely related through the effects produced by each force. For example, fractures caused by thermal forces increase rock permeability, and consequently the rate of convective heat transfer is increased. Host rocks are heated faster and pluton is cooled faster in these fractured regions; the strain rate increases and causes more fractures. Each of the transport processes is coupled in a similar manner and subject to analysis by the concept of “feedback” systems (Forrester, 1968; Norton and Knapp, 1979).

Many pluton-host rock systems were formed by a magma, intruded to an unknown depth, that had one horizontal dimension greater than another and a vertical dimension of uncertain magnitude. The plutons are fractured to various degrees, but in general silicic plutons have a paucity of features indicative of magma convection. Fractures are often continuous into the host rocks, and both regions display a range of orientations, continuation, and age relationships.

Stress evolution and crystallization history are analyzed below for an idealized, two-dimensional single pluton-host rock system subsequent to intrusion. The pluton is approximated by a semi-infinite dike. The initial condition is a magma body cooling in the upper crust. It is assumed that the magma was emplaced rapidly relative to the rate of heat transfer from the body (Norton and Knight, 1977). From these conditions the
changes in stress and magma pressure caused by evolving gravitational, thermal, and magmatic forces are computed. Fracture failure within solid portions of the system is defined as the condition when stress values coincide with corresponding values of the failure envelopes. Feedback between stress and magma pressure is explicitly represented in the governing equations, as is feedback between fracturing and magma pressure. No attempt is made to determine stress redistribution nor permeability variations caused by fracturing.

Analysis of the thermal and mechanical processes in this manner permits a subdivision of the problem into two distinct and independent parts: (1) analysis of conductive cooling — the thermal history, and (2) analysis of mechanical energy transport — crystallization and stress evolution — as a consequence of the conductive cooling. The temperature distribution within the pluton and host rocks is computed initially for the time duration of the system. Then temperatures at discrete times are used to compute the mechanical energy effects.

**MATHEMATICAL REPRESENTATION**

**Notation and Conventions**

All pertinent notation has been compiled below to abbreviate equation development. All italicized variables indicate vector or matrix quantities, and a superscript * on these quantities denotes transposition. Throughout this study compressive stresses are defined to be negative, and magma is used to refer to the bulk, solid–liquid–vapor magma system.

\[
\begin{align*}
A &= \text{area (m}^2) \\
A^* &= \text{Helmholtz Free Energy (J)} \\
C &= \text{line integral} \\
D &= \text{nodal displacement vector (m)} \\
E &= \text{matrix containing elastic constants; transforms strains to stresses (bar)} \\
F_e &= \text{external surface force (bar)} \\
F_n &= \text{nodal force vector (nt)} \\
g &= \text{gravitational acceleration vector (m sec}^{-2}) \\
P &= \text{pressure (bar)} \\
P_0 &= \text{initial pressure (bar)} \\
R &= \text{denotes integration over a region in space} \\
r &= \text{displacement vector (m)} \\
S &= \text{denotes surface integral} \\
S^* &= \text{total entropy of a system (J}^\circ\text{C}^{-1}) \\
T &= \text{temperature (°C)} \\
T_0 &= \text{initial temperature (°C)} \\
T &= \text{nodal temperature matrix (°C)} \\
T_{u,D} &= \text{transformation matrix relating nodal values to a value at a point within the element} \\
T_{\Delta v,D} &= \text{transformation matrix relating nodal displacements to elemental volume changes (m}^2)
\end{align*}
\]
processes related to magma crystallization and stress evolution

\[ T_{\epsilon,D} = \text{transformation matrix relating nodal displacements to strains (m}^{-1}\) \\
\[ t = \text{time (sec)} \]
\[ u = \text{displacement in x-direction (m)} \]
\[ V = \text{volume (m}^3) \]
\[ \alpha_L = \text{coefficient of linear thermal expansion (}^\circ\text{C}^{-1} \) \\
\[ \epsilon = \text{strain matrix} \]
\[ \epsilon_0 = \text{initial strain matrix} \]
\[ \zeta = \text{potential energy (J)} \]
\[ \nu = \text{Poisson's ratio} \]
\[ \rho = \text{volume averaged density of a system (km}^{-3} \) \\
\[ \sigma = \text{stress matrix (bar)} \]
\[ \sigma_1 = \text{incremental stress matrix (bar)} \]
\[ \sigma_\text{max} = \text{maximum principal stress (bar)} \]
\[ \sigma_\text{min} = \text{minimum principal stress (bar)} \]
\[ \sigma_0 = \text{initial stress matrix (bar)} \]
\[ \omega = \text{work (J)} \]

**Thermal History**

Transport of thermal energy in pluton-host rock systems is caused by both conductive and convective processes. Convective circulation of hydrothermal fluids is an inevitable consequence of magma intrusion into the crust; the magnitude of this circulation is directly related to the rock permeability. Although the conditions of fracture failure for a high temperature crystalline rock are poorly known, the best evidence available suggests that igneous rocks fracture slightly below their solidus temperatures (Norton and Taylor, 1979). Heat transfer from these magmas is essentially limited by conduction.

Conditions outside the pluton are generally dominated by convective heat transfer because of the relatively large initial permeability of many crustal rock types. However, geologic observations indicate that the permeability of these rocks has been significantly augmented by fractures formed during the cooling of nearby intrusions.

Therefore, the crystallization and stress history of a magma and the formation of fractures that augment host rock permeability are examined for a system in which the heat transfer is only by conduction. The governing equations for heat transfer by pure conduction are solved numerically using a finite difference approximation to the equations as described by Norton and Knight (1977) to obtain temperature distributions in the system for a sequence of times throughout its history. These temperatures are then used to compute the crystallization history and stress evolution of the system. The limitations of these assumptions are evaluated in the concluding sections.

**Magma Crystallization and Stress Evolution**

The crystallization of magma and stress evolution in hydrothermal systems can be described by the change in Helmholtz Free Energy

\[ dA^* = -S^*dT - d\omega - d\zeta \]

(1)
As previously stated, the idealization that fracturing in the system produces no change in permeability permits the thermal and mechanical processes to be considered separately. Consistent with this idealization is the assumption that mechanical work produces no significant change in temperature. However, this does not preclude analyzing the effects of thermal changes in the system on conditions of stress or magma pressure. Thus as the system evolves from one equilibrium thermo-mechanical state to another

$$0 = \int_{c} dA^* = - \int_{c} d\omega - \int_{c} \int_{r} \int_{s} \left( \rho \int_{c} g \cdot dr \right) dV \quad (2)$$

where the potential energy contribution, $d\xi$, to the total energy has been expanded in terms of the gravitational potential energy. Eq (2) is applicable only to regions within which there are continuous variations in the intensive thermodynamic variables. In hot pluton environments there may be two such regions: solid regions that include host rocks and completely crystalline portions of the pluton and fluid regions that include partially molten portions of the pluton. Therefore, separate governing equations identical to (2) are required for each region; the equations differ only in the work term, because this term depends on the mechanical response of the material.

**Solid-elastic regions.**—The mechanical response of completely crystalline portions of the system depends primarily on confining pressure, differential stress, temperature, and strain rate. For hot pluton systems in the upper crust, the confining pressure and magnitude of the differential stress is less than 3 kb; temperatures in the solid portions are less than 800°C. Under these conditions and for strain rates greater than about $10^{-5}$ s$^{-1}$, rocks are considered to have a linear response (Griggs, Turner, and Heard, 1960; Heard, 1967; Handin, 1966; Jaeger and Cook, 1976). Therefore, the solid portions of the system are treated as linear elastic material. Deviations from a linear response for lower strain rates would mean that strains predicted in this study are minimums and fracture abundances are maximums, because intracrystalline gliding would tend to be more important than fracture failure.

The work term in (2) for solid-elastic regions is expressed (Gibbs, 1961) by

$$- \int_{c} d\omega = \int_{c} \int_{r} \int_{s} \varepsilon \sigma dV - \int_{c} \int_{s} \left( \int_{c} F_{\sigma} dr \right) dA \quad (3)$$

where the first term on the right is strain energy, and the second is work caused by tractions on discrete regions. Expanding $\sigma$ in (3) for a linear elastic material and substituting the result into (2) gives the governing equation for equilibrium, isothermal mechanical processes in closed, linear elastic regions.
\[ 0 = \iiint_R \varepsilon \sigma_i \, dV + \iiint_R \varepsilon \sigma_0 \, dV - \iiint_R \int_{s}^{c} F_e \cdot dr \, dA - \iiint_R \rho \int_{s}^{c} g \cdot dr \, dV. \] (4)

Stresses caused by tectonic processes are incorporated through the initial stress term, \( \iiint_R \varepsilon \sigma_0 \, dV \); effects caused by gravitation are expressed through the last term; forces on the solid portions of the system caused by an enclosed fluid magma body are transmitted across the surface separating the two phases and thus are expressed through the surface integral. Thermomechanical effects are incorporated into (4) by transforming thermal changes into initial stresses via the elastic thermomechanical parameter, \( E \alpha \),

\[ \sigma_0 = \int E \alpha \, dT \] (5)

The term containing initial strains, \( \varepsilon_0 \), in (4) accounts for previous deformations and is not considered further. Material anisotropy can be expressed through \( E \), which expresses strains as a function of stresses and contains the elastic parameters. In this study the solid regions are assumed to be isotropic. Incremental stresses, \( \sigma_i \), and the strains in (4) are dependent variables.

Nonlinear variations in material elastic properties as well as the other parameters require that eq (4) be evaluated by discrete methods. The finite element method provides a stable numerical solution to eq (4) (Zienkiewicz, 1971). To obtain this solution, the domain was subdivided into \( n \) subregions, finite elements, over which the thermodynamic variables and displacements vary according to a specified function. Accuracy of the results then depends directly on how closely these specified functions approximate the real situation. Material properties and resultant stresses are averages over these subregions.

The incremental stresses and strains in terms of displacements are expressed by \( n \) equations of the form (Zienkiewicz, 1971):

\[ 0 = K_E \cdot D + \iiint_R T_{\varepsilon,D} \cdot \sigma_0 \, dV - \iiint_R T_{\varepsilon,D} \cdot E \cdot \varepsilon_0 \, dV \]

\[ - \iiint_S T_{\varepsilon,D} \cdot F_e \, dA - \rho g \iiint_R T_{\varepsilon,D} \, dV. \] (6)

The elastic stiffness matrix which transforms displacements into forces, \( K_E \), is defined to be (Zienkiewicz, 1971):
\[ K_E = \iiint_{V} T_{\tau_e,D} \cdot E \cdot T_{\epsilon,D} dV. \] (7)

The displacements along the boundary of each element, \( D \), are the unknown in (6). \( T_{\tau_e,D} \) is a matrix relating \( D \) to the strain within an element. It is derived from the definition of strain and the specified functions describing displacement variations. The \( n \) equations of the form (6) are solved simultaneously to obtain the displacements, stresses, and strains at points within the system. The resulting stresses and strains define the thermodynamic state with the lowest possible Helmholtz Free Energy.

**Fluid regions.**—As a magma cools to subsolidus temperatures, the mechanical response of the mixture of solids, liquids, and gases varies from viscous to elastic-plastic to linearly elastic (Murase, 1962; Shaw, 1965, 1969, 1979; Shaw and others, 1968; Bottinga and Weill, 1970; and Murase and Mc Birney, 1973). Only the end members of this sequence are considered, however, because there are few quantitative determinations of the temperature and pressure boundaries between these various regions and values of the elastic-plastic parameters. Therefore, magma will be treated as an ideal, homogeneous fluid which can be described by isotropic pressure. The magma solidus is arbitrarily chosen as a discontinuous boundary between ideal fluid and linear elastic behavior. As a portion of the pluton attains solidus conditions the stress is taken to be isotropic and equal to the pressure but can subsequently become anisotropic. The magma is also treated as a hydrostatic body as a further simplification. This seems a reasonable first approximation because of the lack of magmatic convection within silicic plutons. Convection would have a significant effect on this computation only when it would redistribute energy on the same order of magnitude as conductive, gravitational, and work processes. If the contribution were significant, the result would be smaller stresses and fewer fractures than predicted by this study.

Equations describing the above fluid model were developed along with solution methods that were compatible with existing finite element methods. Because they are the first published attempt to solve this problem, these equations and methods are presented in some detail.

For homogeneous fluids the work term in (2) is expressed by

\[ -\int_c d\omega = -\int_P dV - \int_S \left( \int_{c} \left( F_e \cdot \delta r \right) d\mathbf{A} \right). \] (8)

Pressure is defined by the integral of its total differential

\[ P = P_0 + \int_V \left( \frac{\partial P}{\partial T} \right)_V dT + \int_V \left( \frac{\partial P}{\partial V} \right)_T dV \] (9)

where the fluid thermomechanical parameter, \((\partial P/\partial T)_V\), transforms thermal changes into pressures and is the analog to the elastic solid thermomechanical parameter, \(E\alpha_T\). Substituting (9) into (8) and the result into
(2) gives the equation for an equilibrium isothermal, mechanical process in a closed hydrostatic region

\[
0 = -P_o \int dV - \int \int \int \left( \frac{\partial P}{\partial V} \right)_T dV dV - \int \int \int \left( \frac{\partial P}{\partial T} \right)_V dT dV
- \int \int \int \int \left( \sum \int F_e \cdot dr \right) dA - \int \int \int \int \left( \rho \int g \cdot dr \right) dV. \quad (10)
\]

Eq (10) is transformed into the force-displacement finite element form by completing the line integrals and requiring small perturbations from equilibrium; then,

\[
0 = -\Delta V \left[ P_o + \left( \frac{\partial P}{\partial T} \right)_V \Delta T \right] - \Delta V \left( \frac{\partial P}{\partial V} \right)_T \Delta V
- \int \int \int \Delta r \cdot F_e dA - \rho g \int \int \int \int \Delta r dV. \quad (11)
\]

The change in volume is related to nodal displacements by

\[
\Delta V = D \cdot T_{\Delta V,D} = D \cdot \nabla \cdot T_{u,D} \quad (12)
\]

and substituting (12) into (11) gives

\[
0 = -T_{\Delta V,D} \left[ P_o + \left( \frac{\partial P}{\partial T} \right)_V \cdot T_{u,D} \cdot \Delta T \right] + K_F \cdot D
- \int \int \int T_{\Delta V,D} \cdot F_e dA - \rho g \int \int \int \int T_{\Delta V,D} dV \quad (13)
\]

where \( K_F \), the analog to the elastic stiffness matrix, is defined to be

\[
K_F = -T_{\Delta V,D} \cdot \left( \frac{\partial P}{\partial V} \right)_T \cdot T_{\Delta V,D}. \quad (14)
\]

The first term on the right in eq (13) accounts for energy changes due to an initial pressure and thermal changes within the magma. Thermal changes affect both the state of the magma and its thermomechanical parameter, \((\partial P/\partial T)_V\). Eq (13) affords for both pure thermal expansion-contraction and phases changes. The fluid thermomechanical parameter is the most important term in eq (13); it transforms thermal and resultant phase changes into pressures. In a cooling magma chamber the sign of this contribution to pressure depends on the sign \((\partial P/\partial T)_V\); a negative value, which occurs only when the magma boils, demands a pressure increase upon cooling. This will be discussed in detail later. The gravitational term contributes to the initial pressure distribution and describes buoyant forces within the magma. It is indirectly affected by thermal changes through \( \rho \). The surface integral couples adjacent finite
elements and thus explicitly affords for mechanical interaction between regions.

The effects of initial pressures caused by the magma’s emplacement history, thermal changes, and gravitational forces are propagated throughout the system, and the resultant displacements are such that the Helmholtz Free Energy of the magma is minimum. The final pressure in a fluid element is computed from the equation of state for the fluid material. Both temperature and fluid density are known and require a unique pressure at equilibrium.

Eq (14) is a symmetric transformation, and therefore $K_P$ is required to be symmetric. Thus the fluid elements are completely compatible with the elastic elements. The symmetry of $K_P$ would appear to distinguish this formulation from one given in Knapp and Knight (1977), but a close examination of the latter stiffness matrix shows that it, too, is symmetric, although it is not required to be.

To suppress shear stress and impose the hydrostatic no-flow condition, it is necessary to use a composite quadrilateral membrane element, constructed by superimposing two sets of two constant strain triangular fluid elements in the composite. This formulation for fluid elements was tested by solving the analog to the frictionless piston experiment commonly employed in thermodynamics; differences between the numerical approximations and analytical thermodynamics are less than $\pm$ 5 percent.

THE SYSTEM

The mathematical basis for examination of isothermal mechanical processes in hydrothermal systems is expressed in eqs (4) and (10). Equations identical to (4), written for $n$ linear elastic elements, and equations identical to (10), written for $m$ fluid elements, are solved simultaneously. The dynamic interaction between magma and host rocks is accounted for through contiguous elastic and fluid elements. Thus, forces resulting from variations of thermodynamic states in any region propagate throughout the system such that equilibrium is maintained between and within magma and host rocks.

To solve (4) and (10), elastic and fluid material properties representative of common crustal rocks and magmas are required. The properties of crystalline albite, as derived from its equation of state (Burnham and Davis, 1969), are used for the elastic parameters. The properties of a tonalite magma with 2 wt percent $H_2O$, as approximated by the albite–$H_2O$ magma system (Burnham and Davis, 1969), using Whitney’s (1977) phase boundaries for the tonalite magma, are used for the fluid parameters. The details of the derivation of these parameters can be found in Knapp (ms).

The fluid thermochemical effects are the most important effects in determining stress evolution and crystallization history. They vary drastically in sign and magnitude between different phase regions (fig. 1). In the two phase, S–L, region $(\partial P/\partial T)_V$ is always positive and varies between about 50 and 90 bars/°C. Thus cooling within this phase region is
a drive toward decreasing magma pressure, at least 500 bars for every 10°C cooling interval. However, \((\partial P/\partial T)_v\) becomes negative as the magma cools, becomes saturated with H_2O, and develops a discrete vapor phase. The magnitude also greatly increases so that cooling within the three phase, S–L–V, region is a large drive toward increasing magma pressure: at least 1 kb for every 10°C cooling interval.

A range of coexisting pressures and temperatures occur throughout a magma body because of its relatively great lateral and depth extent, and thus the magma may not fall entirely within either the S–L or S–L–V phase regions. The existence of S–L–V phase regions within the magma does not insure an increase in magma pressure; the drive toward increasing pressure in the region must be compensated with the drive toward decreasing magma pressure from coexisting S–L regions. However, when a sufficiently large volume percent of the magma is boiling the pressure increases. In the system modeled here this value is about 8 percent, but, as can be seen from the P–T variations of the fluid thermomechanical parameter (fig. 1), this value is highly dependent on the system's configuration. A pluton at shallow depths and of small depth extent may rarely experience a large increase in magma pressure; the magnitude of \((\partial P/\partial T)_v\) in this region is quite small, less than 5 bars/°C, requiring almost the entire magma to be boiling for pressure to increase.

It should also be noted that the magnitude of \((\partial P/\partial T)_v\) is highly dependent on the water content of the magma. A greater water content

Fig. 1. Fluid thermomechanical parameter for a hydrous tonalite magma with 2 wt percent H_2O. The sign and magnitude of the fluid thermomechanical parameter, \((\partial P/\partial T)_v\), is dependent on the phase region. In the two-phase (S–L) region it is positive and about 60 bars/°C; a 10°C cooling increment is a drive toward a 600 bar magma pressure decrease. In the three-phase (S–L–V) region, \((\partial P/\partial T)_v\) has a large magnitude and is negative; a 10°C cooling increment is a drive toward a 1 kb magma pressure increase. The computations also afford for changes in magma pressure due to changes in volume of the magma chamber and due to spatial variations in the sign and magnitude of the fluid thermomechanical parameter. The magnitude of \((\partial P/\partial T)_v\) directly depends on the initial H_2O content of the magma. For details of data construction see Knapp (ms).
than the present 2 percent value would increase the magnitude of the
thermomechanical parameter in the S–L–V region requiring less of the
magma to be boiling before magma pressure increased.

The fact that the magma is not a constant volume system is explicitly
afforded for in the calculations. Thermomechanical parameters are used
to compute pressures — and stresses — at the beginning of a time step.
These are inserted into the appropriate location in the governing equa-
tions, whereupon solution of (4) and (10) gives the final equilibrium
pressure/stress distribution.

The most important assumptions involved in eqs (4) and (10) include
using the solidus as a discontinuous boundary between fluid and linear
elastic material behavior, treating the magma as a hydrostatic body,
treating the magma as a closed system, and neglecting convective heat
transfer in the solid portions of the system. Treating the magma as a
hydrostatic body assumes that convection within the magma does not
significantly contribute to the total Helmholtz Free Energy. Consequent-
ly, the predicted magma pressures and subsequent timing of all events
are affected, because of the effect of convection on the thermal term in
(10) as well as on diminishing of pressure gradients within the pluton.

Treating the magma as a closed system neglects depletion of the
magma chamber by volcanic eruptions, additions to the magma chamber
from both subsequent intrusions and stopped blocks, and effects due to
continuous connection with a larger magma chamber at depth. Eruption
presumably occurs when $P_{\text{magma}}$ increases to approximately $\sigma_r$, and the
roof rocks fail. Inflation of the magma chamber by injections of magma
increase magma pressure and temperature and tend to increase fracture
abundance by lengthening the thermal history of the system, whereas the
effect of convection within a magma chamber is to maintain magma pres-
sure at a constant value.

**COMPUTATIONAL MODEL**

The mathematical representation of the conceptual model, eqs (4)
and (10), forms the basis for numerically examining crystallization history
and stress evolution in a hot pluton environment, where heat transfer is
by pure conduction. Initial and boundary conditions used in this pre-
liminary study were chosen so that the most effective processes could be
identified and their interactions understood.

The system analyzed is a pluton, 3 km wide by 4.5 km high and
infinite in the third dimension, at a depth of 4.5 km in an 18 km by 9 km
section of upper crust. The thermal history, crystallization history, and
stress evolution are computed for a two-dimensional cross section through
the semi-infinite dike.

**Thermal History**

Heat transfer in the impermeable system depends on pluton geomet-
try and heat content, transport properties of phases, initial thermal
conditions and heat transfer boundary conditions (fig. 2). Emplacement
of the hydrous tonalite magma is assumed to occur instantaneously, so
that initial host rock temperatures and pluton heat content (367 kcal/km depth) are undisturbed. A two-dimensional symmetrical system is discretized into 400 evenly-spaced grid points; temporal and spatial errors introduced by this finite difference discretization are less than 10 percent (Norton and Knight, 1977). Temperatures at the grid points are computed for discrete times for a period of $5 \times 10^6$ yrs and are used in (5) and (12) to compute the thermodynamic state of the magma and the equilibrium stress distribution.

**Crystallization History and Stress Evolution**

Crystallization history and stress evolution in the heuristic system depend on initial geometries, pressures, stresses, and material properties, as well as temperature distributions. Geometric similarity between finite difference domain in the thermal calculations and finite element

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**Fig. 2.** Thermal initial and boundary conditions and isotherms at select times (in thousands of yrs) after pluton emplacement, for the pure conductive cooling model used in this study.
domains in the stress calculations is maintained by using initial conditions similar to those in the thermal problem. Therefore, a magma body is assumed to be emplaced instantaneously in a crustal environment that is characterized by an isotropic stress field equal to the lithostatic load. Initial magma pressure is defined by the equilibrium conditions between stresses, magma pressure, and body forces.

The initial stress field is slightly perturbed from lithostatic conditions because of the equilibrium conditions with magma pressure, which has an initial value of about 1.5 kb at the top of the pluton and varies by ρ(T,P) • g downward. Differences between pressure and stress also initially increase downward, because the density of hydrous tonalite magma is always less than that of the host rock. The initial P–T conditions are such that the magma contains two phases, except for a small chilled margin approx 150 m thick, which is the result of approximating the system by a discrete grid.

The system is represented by a two-dimensional domain, but, since the domain is symmetric, only half of it is used. This half-domain is discretized into 343 elements, defined by 183 nodes, with two degrees of freedom per node. Constant strain, triangular membrane elements are used in the elastic regions, and composite quadrilateral elements, described earlier, are used in the fluid regions. Resolution of the results is greatest where element density is greatest, which in this study is in and immediately around the pluton. The bottom boundary is constrained from vertical and horizontal displacements only; there are no displacement restrictions on the top boundary.

Numerical solution of the governing equations determines equilibrium values of magma pressure and rock stress for any given conditions of temperature. The temperature distribution in the system analyzed is computed independently as a function of time, and, therefore, eqs (4) and (10) are implicit functions of time through the thermomechanical parameters, (5) and (10). Equilibrium pressures and stresses are computed incrementally as a function of time by solving a series of initial and boundary value problems. Pressure and stress values from the current time and temperatures from the subsequent time define initial conditions in (4) and (10) and permit calculation of new pressure and stress values.

Material properties required to solve eqs (4) and (10) are calculated at the beginning of each time step and therefore lag half a step behind actual conditions. These properties are calculated for current values of temperature and for previous pressure values. Because magma properties are highly temperature- and pressure-dependent, numerical integration limits are restricted to ΔT = −5°C to ensure convergence of the numerical equations to within 10 percent of the true value.

**Computational Results**

**Thermal History**

The host rock thermal regime is characterized by propagation of isotherms away from the pluton in the early stages of the cooling history,
followed by their descent back toward the pluton and regional values in the later stages (fig. 2). Points within the host rocks experience an initial temperature increase followed by a slow exponential decay; the net rate of decay attains a maximum of about $2 \times 10^{-4} \degree C/yr$ near the pluton (fig. 3). The rate of temperature increase, magnitude of the temperature maximum, and the initiation of temperature decrease are inversely proportional to the distance away from the heat source.

The plutonic thermal regime is characterized by continually decreasing temperatures; the rate of decay decreases with distance from the pluton-host rock boundary (fig. 3).

**Crystallization History**

The redistribution of thermal energy in the system is the most important cause of temporal variations in stresses and state of the magma. The style of magma crystallization depends primarily on the sign and

![Diagram](image)

*Fig. 3. Temperature variations for positions within the host rocks and pluton. Locations are denoted by numerals in figure 2.*
Fig. 4. Locations and geometry of the various phase regions within the pluton. Numerals denote times in thousands of yrs. The chilled margin of the pluton is an initial condition; the adiabatic regions denote locations at which magma pressure induced fracturing in adjacent rocks, which is accompanied by a rapid pressure drop and crystallization of the magma; crystallized regions form by any other approach to the solidus; the three-phase region is the S−I−V region, and the two-phase region is the S−L region. Multiple magma chambers are formed by 17,000 yrs.
magnitude of the fluid thermomechanical parameter, the volume fraction of vapor-saturated regions within the magma, and the rate of thermal changes.

Three-phase regions are initially confined to the top corners of the pluton, but then they gradually expand throughout the upper part of the magma and downward adjacent to the flanks of the pluton (fig. 4). Complete crystallization occurs soon after separation of a vapor phase and follows the three-phase region progressively down the pluton flanks.

Fractures caused by incremental increases in magma pressure result in adiabatic crystallization between 16,000 and 17,000 yrs (fig. 4) and in the formation of three discrete magma chambers. Adiabatic crystallization results from the direct dependence of rock failure and fluid pressure. As rocks adjacent to magma fracture, pressure decreases within the magma. This pressure decrease could be evident as igneous dikes or could be the release of H₂O — especially if the adjacent magma is boiling. Decreasing pressure shifts the system toward crystallization because of the shape of the magma solidus; therefore, when failure conditions were attained adjacent to a fluid element, that element crystallizes adiabatically, and the pressure drop propagates throughout the system.

Multiple magma chambers are caused by density variations arising from nonlinear pressure changes with depth and the effect of fracturing on the crystallization process. Regions that have crystallized adiabatically can segment the magma chamber into several discrete chambers, particularly at the depths where rocks are close to failure conditions. The deepest chamber is clearly a result of these factors (fig. 4).

The formation of multiple magma chambers appears to be a characteristic feature of the crystallization process and should cause composite plutons to form with chilled margins only along the host rock-pluton contact. Furthermore, the individual chambers evolve in a distinctive and predictable style totally independent of the other chambers except for minor mechanical interactions through the septa. Because the upper chambers are relatively small, their crystallization paths are nearly constant volume paths (fig. 5, path 2), that is, the actual pressure evolution is closely approximated by integrating the fluid thermomechanical parameter over the computed temperature change. Pressures first decrease because of the lack of vapor saturation in this chamber (fig. 4), but when the three-phase region is reached, pressure rapidly increases until the solid envelope fractures and adiabatic crystallization occurs. It is important to realize, however, that adiabatic crystallization of the magma in the chambers does not occur until about 5000 yrs after they become isolated. The deepest of these chambers evolves differently because it is much larger than the others; its crystallization path is not a constant volume path (fig. 5, path 3).

The crystallization history of magma within the model system depends on rates of cooling and pressure change (fig. 5). Cooling continuously moves the system toward the solidus; those regions closer to the pluton-host rock contact cool and crystallize more rapidly. Pressure in-
Fig. 5. Computed crystallization paths for various points in the model system. Tic marks are for every 5000 yrs. Stars denote conditions at which magma pressure increased sufficiently to cause fracturing in adjacent rocks and subsequent adiabatic crystallization of the magma in that region. Each dramatic change in slope corresponds to times at which the volume fraction of boiling increased above or decreased below 8 percent.
creases generally move the magma conditions away from the solidus, but
the failure of the confining solid carapace around the magma relieves
pressure and crystallizes adjacent magma. This reduces both the amount
of boiling and the magma pressure, consequently P–T paths move toward
the solidus until the cycle is repeated. Pressure variations reach a maxi-
mum of 500 bars, are discontinuous, and may be either positive or nega-
tive. They tend to increase rapidly when an 8 volume percent of magma
is boiling in this system.

Textural, structural, and composition differences within and be-
tween individual magma chambers and between magma chambers and
previously solidified parts of the pluton should be apparent in natural
analogs to these systems. Examples of this type of crystallization history
include the Rocky Hill stock (Putnam and Alfors, 1969), Santa Rita
stock (Nielsen, 1968), as well as other localities reported by Balk (1987)
and observed by our own field studies.

**Stress Evolution**

Static magma bodies in the crust cause a radial and concentric stress
field in which the maximum principle stress is in the radial position (fig.
6). This stress geometry typically occurs in an elastic domain where the

![Diagram of stress trajectories and temperature changes](image)

**Fig. 6.** Principle stress trajectories and net change in temperature (°C) in the
model system at 0, 35,000, and 150,000 yrs after pluton emplacement. Trajectories are
radial and tangential around the cooling center within the pluton.
internal pressure is less than the applied stress (Odé, 1957; Robson and Barr, 1964; Roberts, 1970; Timoshenko and Goodier, 1970). The stress field caused by magma pressure is relatively short-lived because calculations indicate the magma is completely solidified by 32,000 yrs. However, differential heat transfer between pluton and host rocks continues to produce thermal stresses long after complete crystallization of the magma.

Maximum principle thermal stresses are deflected toward regions where the magnitude of $\partial T/\partial t$ is greatest (eq 5); in the model system, this is a cooling center located at the top of the pluton (35,000 yrs after emplacement). Within the host rocks $\sigma_1$ trajectories are radial toward the cooling center, but within the pluton $\sigma_1$ becomes concentric (fig. 6). As the pluton cools, the region of greatest temperature change expands downward into the pluton, deflecting the trajectories as it moves so that the radius of curvature of $\sigma_3$ trajectories has greatly increased by 150,000 yrs. Although trajectories are now nearly vertical through the center and converge at the upper pluton corner, $\sigma_1$ still radiates toward the enlarged cooling region. This geometry reflects the elongate nature of the cooling center and its localized extreme at the corner. It is important to note that $\sigma_1$ is still nearly horizontal in the top center portions of the pluton.

Evolution of stress in hydrothermal systems depends on variations in temperature and magma pressure. Differential heating, in general, induces compressive stresses; differential cooling induces tensile stresses. Because heating and cooling regimes are contiguous in pluton environments, their mechanical effects interact. Temperature increases in the host rocks cause expansion away from the magma chamber, whereas cooling within the pluton results in contraction. Therefore, a tensile type stress is generated normal to the pluton-host rock boundary. The effect is that cooling increases (decompresses) the principle stress, but heating the host rocks decreases (compresses) the concentric stress, $\sigma_3$, and increases the radial, or normal stress, $\sigma_1$ (fig. 7).

![Diagram](image_url)

**Fig. 7.** General effects of thermal changes and magma pressure increases in stress development in the model system. Thermal effects are for times at which magma pressure is decreasing or when there is no magma left in the system. Magma pressure effects are superimposed upon thermal effects. Tensile stresses are positive.
Magma pressure variations perturb these trends both within the host rocks and within the pluton (fig. 7). Magma pressure effects are episodic and are superimposed upon continuously acting thermal effects in figure 7. This superposition causes the general trend of stress evolution to be more complex than that predicted by solely considering magma pressure variations (Timoshenko and Goodier, 1970). Magmatic effects can be greater than, equal to, or less than thermal effects, depending on elapsed time and distance from the remaining magma body. An increase in magma pressure causes the radial stress ($\sigma_1$) in the host rocks to become more compressive and somewhat lessens the rate of decrease of the concentric stress ($\sigma_3$). An increase in magma pressure mainly affects the concentric stress ($\sigma_3$) within the crystalline portions of the pluton; $\sigma_3$ becomes more compressive. The effects of magma pressure variations are insignificant at locations more than about 1 km away from the magma, because pressure dissipates as the inverse square of distance.

Stress varies as a function of time because of magma pressure variations (fig. 8, path 1), where each deviation in trend corresponds with

![Graph showing stress paths](image)

**Fig. 8.** Computed stress paths at selected locations within the model system, showing the combined thermal and pressure effects. Each change in slope corresponds to times at which the volume fraction of boiling has increased above or decreased below 8 percent. Arrowheads are at 5000 yr intervals through the magmatic history at each point. The next time denoted after crystallization is 100,000 yrs. The hydrostatic stress condition, $\sigma_1 = \sigma_3$, is shown for reference. Stress redistribution due to fracturing is not afforded for, and as a result stresses during the later history of the system develop geologically unreasonable values.
changes in sign of \(\frac{\partial P}{\partial t}\), until the pluton is completely solid at 32,000 yrs. Maxima and minima in path 2, located within the host rocks, also correspond to changes in \(\frac{\partial P}{\partial t}\), until (~20,000 yrs) when the magma has receded to a distance greater than 1 km away from the point, and further variations in magma pressure do not affect stress evolution at this location. The decompression of \(\sigma_1\) and \(\sigma_3\), beginning at 100,000 yrs, coincides with the initiation of cooling at this host rock location.

Stress paths are also a function of rapid cooling or heating, and the tops of plutons are regions of greater rates of stress change than any other regions (fig. 8, compare paths 1 and 3). Path 3 is located at the top of the pluton and undergoes extremely rapid cooling for the first 35,000 yrs; each segment of path 3 is, therefore, longer than the corresponding segment of path 1. For instance, in path 3, \(\sigma_1\) increases by about 1200 bars and \(\sigma_3\) increases by about 200 bars from \(t = 0\) to 1000 yrs. During the same period, \(\sigma_1\) of path 1 increases by only 600 bars, and \(\sigma_3\) does not change. Stress variations with respect to time depend on magma pressure variations, proximity to the magma chamber, and the local \(\frac{\partial T}{\partial t}\).

**Fracture Evolution**

Fracture failure in hot pluton environments is a result of stress variations caused by pore-fluid, magma, and thermal forces. The timing, location, and geometry of fractures produced by these processes can be estimated from the computed stresses and a constitutive equation for the solid, using a modified Coulomb-Naviere theory (Paul, 1961; Jaeger and Cook, 1976) as a first approximation to brittle failure (fig. 9). Though the three fracture producing forces often are spatially and temporally concurrent, it is important to examine magma pressure and thermal processes separately. The consequences of pore-fluid expansion were ex-

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**Fig. 9.** Brittle failure regimes used in this study: a modified Navier-Coulomb criteria after Paul (1961) and Jaeger and Cook (1976), using a crushing strength, \(C_c\), of ~1 kbar and a tensile strength, \(C_t\), of 50 bar.

**Fig. 10.** Computed locations of magma pressure fracturing in the model system. Numerals denote times in thousands of years. Magma pressure fracturing in the model system occurs in the rocks around the margins of the magma, when magma pressure increases induce tensile failure.
amined by Knapp and Knight (1977). Stress paths move continuously through failure regimes, because stresses were not redistributed after fracture failure. This approach is consistent with Odé (1957), Cummings (1968), and Koide and Bhattacharji (1975) in correlating predicted stress orientations and fracture geometry.

Fracture caused by increasing magma pressure.—Fractures caused by increasing magma pressure form periodically throughout the 82,000 yr magmatic history (fig. 10). Within the initial several thousand years after emplacement, fractures are restricted to a region at the top of the pluton, but as cooling continues conditions for fracture formation occur at greater depths until at 20,000 yrs these conditions extend throughout most of the pluton. Fractures form also as a consequence of vapor saturation in isolated magma chambers and are restricted to the boundaries of these chambers. A composite map of regions where fracture failure criteria are realized illustrates three major conclusions:

1. The flanks of the pluton tend to be unaffected by increased magma pressure because of the relatively thick layer of solid developed there during periods of low magma pressure.
2. Large portions of the isolated magma chambers are unfractured.
3. Some regions experience multiple fracturing events.

Fracture failure caused by increasing magma pressure occurs only by tension; therefore, the directions of the minimum principle stresses coincide with the strike directions of fractures. Tensional failure also implies that these types of failure could be accompanied by dike injection.

The pluton has been subdivided into three failure domains, and histogram rosettes of dips of the fracture planes have been plotted for each domain (fig. 11). The relative abundance of the various fracture orientations depicted by histograms is based only on the number of fracture failure events each finite element has experienced. Material in an element is assumed to have fractured when its maximum principle stress is less than the pressure in an adjacent magma element plus the tensile strength of the rock. If \( \sigma_1 \) is exceeded by the pressure in two adjacent magma elements, then two fracture events are recorded. Orientations of the fracture planes are computed at each event and then averaged over the domain to obtain the dip-histogram rosettes. Thus, the diagrams represent the relative abundance of fracture events that produce fractures of a given orientation.

The dips of fracture planes (fig. 11) vary considerably throughout the pluton, in response to the spatial and temporal evolution of the stress field. The predominant orientation changes from vertical in domain 1, to horizontal in domain 2, to vertical and \( \pm 30^\circ \) in domain 3. The dips of secondary abundance also vary through the pluton: \( \pm 60^\circ \) in domain 1 and vertical in domain 2. The secondary vertical fractures on domain 2 occur mainly toward the center of the pluton.

Fracture caused by thermal stress.—Fracture failure caused by thermal stress begins in the upper portions of the pluton several thousand years after emplacement (fig. 12). This extends across the top of the
pluton and down the side margin and into the pluton interior. The style of failure varies spatially and temporally throughout the pluton; failure occurs both by tension and shear at various locations until about 500,000 yrs, then only tensional failure conditions exist over the entire pluton. Shear failure conditions within the pluton are always followed by tension failure conditions. A solid-elastic element is assumed to have fractured when the stress values coincide with the failure envelope conditions.

Shear failure conditions also develop within the host rocks around the corners of the pluton by 50,000 yrs after emplacement (fig. 12), then expand radially outward, and by 400,000 yrs form a large cupola around

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![Diagram of composite locations of magma pressure fracturing and dip histogram rosette diagrams of magma pressure fracture planes.](image1)

**Fig. 11.** Composite locations of magma pressure fracturing and dip histogram rosette diagrams of magma pressure fracture planes. The flanks of the pluton and portions of the multiple magma chambers do not experience magma pressure fracturing. Dip histogram rosette diagrams represent relative abundance of fracturing events that produce cracks of a given orientation. Vertical fracture-producing events are most common on the upper portions of the system, horizontal fracture-producing events are most abundant in the middle, and both are abundant in the lower portions. Fracturing is systematic, and variations reflect the spatial and temporal development of the stress field. Numerals denote times in thousand years. Both shear failure planes are plotted.

![Diagram of computed state of stress in model system in terms of brittle failure regimes.](image2)

**Fig. 12.** Computed state of stress in model system in terms of brittle failure regimes. Shear denotes that stresses within this region plot within the shear failure regime (fig. 8) and similarly for tensile. The no-failure regime is left blank. Fracturing depicted in this diagram results from thermal stresses. Numerals denote times in thousand years.
the upper portions of the pluton. Tensile failure conditions also develop in this region by 300,000 yrs but are more restricted.

Fractures caused by thermal stresses in hydrothermal systems can be mapped according to three different mechanisms; shear, tensile, and combined shear and tensile failures (fig. 13). Pure shear failure occurs only within the host rocks and forms a cupola around the top of the pluton. Tensile failure occurs only within the pluton within two distinct regions, one on the upper portion and one on the lower portion. Shear followed by tensile failure occurs in both pluton and host rocks, mainly confined to the margins of the pluton, but also in a wide zone across the pluton's mid-section.

Orientation of these fractures is determined by the stress field at the time of failure, as well as by the failure mechanism (fig. 13). Fracture orientations in domain 1 are characteristic of a combined tensile and shear mechanism. The conjugate shear set with dips around 60° and 30° is discernible, but the tensile event dominates in the form of vertical fractures. Fracture failure occurs predominantly by tension in domain 2, and the concentration of dips around 10° and 20° is a consequence of the long period of near-horizontal orientation of \( \sigma_3 \) in this area. Region 3 is a mixed tensile-shear region; consequently, there are three dominant fracture directions. The conjugate set dips at 25° and 35°. The tensile fractures bisect the angle between the conjugate set and dip at about 5°.

Fig. 13. Computed composite of stress history in model system and dip histogram roseite diagrams of dips of thermal fracture planes. Thermal fracturing is both by tensile and shear within the pluton but primarily by shear within the host rocks. Rose diagrams represent relative abundance of thermal fracturing events that produce cracks of a given orientation. Thermal fracturing at any location is assumed to occur when the local stresses cross into a different failure regime. The fracture orientations are systematic with variations reflecting temporal and spatial variations on the stress field during fracturing. Both shear failure planes are plotted.
Fig. 14. Computed stress paths at various locations within the system. Arrowheads are at 100,000 yr intervals. Failure regimes are superimposed for reference. Stress redistribution upon fracturing is not afforded for nor are effects of failure on linear elastic material behavior. Subsequent stress values and fracture orientations are first approximations only.
Fig. 15. Strain rates through time at two locations in the model system. Large early variations at location 2 result from magma pressure variations. The magnitudes of the strain rates are 1 to 2 orders of magnitude lower than those commonly measured at locations of major earthquakes.
The stress paths computed for locations that characterize each fracture regime (fig. 14) provide insight into the formation of fractures. In the host rocks where heating rates are rapid, stress paths are directed toward the shear failure envelope, except for some temporary deviations due to local magma pressure increases (path 2). Temperature increases in host rocks outside the shear failure region are slower (path 1) or may be initially farther from the failure envelope. Consequently, temperatures begin to decrease, and the stress path is deflected away from the failure envelope.

Stress paths within the pluton are controlled by the cooling rate, magma pressure, and the time at which the point becomes elastic. Locations within the upper failure zones, where shear is followed by tensile failure, have such large cooling rates that their stress paths intersect the shear failure boundary during the initial increase in pressure. The combined cooling rates and pressure increases cause shear then tensile failure within the region near the middle of the pluton (fig. 14, path 4). The solidified portions barely intersect the shear envelope prior to failing by tension because increases in magma pressure shift the stress path away from shearing conditions.

Tensile failure in the upper region occurs where temperature changes are moderate (fig. 14, path 5). The lower region of tensile failure forms where solidification occurred late, consequently these regions are subjected to stress paths caused by cooling (fig. 14, path 3).

**Strain Rates**

Time averaged strain rates vary between about $10^{-15}$ and $10^{-16}$ sec$^{-1}$ for most of the history of the system (fig. 15). Because the strain rate is directly proportional to distance from the magma, large fluctuations in the magnitude of the strain rate at point 2 (fig. 15) occur early in the system's history. After the magma is completely crystalline, stress and strain rates depend almost entirely on thermal changes. The maximum computed strain rates coincide in time and space with the regions of highest temperature and may compensate for non-brittle material behavior at these elevated temperatures.

The relatively small strain rates imply that the rocks might deform by plastic creep. Although plastic deformation is noted in small plutons, it is minor. This observation suggests that the laboratory measurements of rock deformation as a function of strain rate are not representative of these systems.

**Discussion**

The evolution of stress in magma-hydrothermal systems is a direct consequence of thermal energy dispersion from the magma. Snapshots of this evolving stress field are preserved in the form of dikes, breccias, fractures, veins, and textural variations in igneous rocks. Correlation of these features with state conditions defined from fluid inclusions and mineral compositions and assemblages should better define the geologic history of natural systems.
Rock failure by the formation of fractures augments permeability and porosity in magma-hydrothermal systems. These open, continuous fractures comprise the permeability and flow porosity (Norton and Knapp, 1977), whereas microfractures connected to the more continuous fractures augment diffusion porosity. Qualitative statements of the temporal and spatial variations of conditions that are conducive to rock failure can be made by computing magma pressure and thermal stress conditions in these systems.

Rock fracture caused by variations in magma pressure initiates in the top portions of the pluton almost immediately after inflation of the magma chamber and then rapidly extends through the rest of the pluton as the magma boundary recedes. Rock fracture caused by variations in magma pressure ceases when the pluton is completely crystalline at 32,000 yrs. However, rock fracture as a result of thermal stresses moves inward from the pluton margins toward the core as both shear and tensile failure events. The time difference between these types of fractures increases with distance from the top of the pluton (fig. 16).

These calculations imply significant temporal and spatial variations in the permeability tensor and porosities within the pluton. These implications are consistent with observations on natural systems that indicate that spatial and temporal distributions of fractures and vein mineralogy are characteristic of many hydrothermal systems (Nielsen, 1968; Tittley, Fleming, and Neale, 1978). The correlation between variations in fracture orientation and permeability and vein chemistry is consistent

![Fig. 16. Schematic diagram depicting time lapse between magma pressure fracturing and thermal fracturing within the pluton. Events are nearly simultaneous at the top of the pluton but become more temporally separated with increasing depth so that near the bottom of the pluton there is nearly a 500,000 yr time lapse.](image-url)
with the fact that these parameters determine the source regions of hydrothermal solutions (Norton, 1978). The systematic characteristics of fractures in space and time calculated in this study indicate that the collection of comparable field data will ultimately lead to a more detailed analysis and better understanding of processes in cooling pluton environments.

There are also multiple fracturing events in the host rocks as a result of pore fluid pressure increases and thermal changes. Pore fluids exist in both isolated pores—which comprise 90 percent of the total porosity in crystalline rock (Norton and Knapp, 1977)—and interconnected pores. As temperature increases, fluid pressure within isolated pores rapidly increases and fracturing ensues (Knapp and Knight, 1977). Further increases in temperature serve to propagate the fracture. Thus, as heat is transported away from the pluton, a fracture front propagates along with it at velocities from greater than 10 to less than 1 cm/yr; fracturing is initiated at the time of emplacement adjacent to the pluton and is occurring at a radius of about 3 to 4 km $300 \times 10^3$ yrs after emplacement for the system considered in this paper. In the interconnected pores, the tendency for temperature increases to increase fluid pressure must compete with the tendency for fluid flow to dissipate lateral gradients in pressures. For a given permeability there is a critical rate of heating at which fluid pressure will increase and ultimately result in fracturing.

Thermal changes in the host rocks initially cause rock failure about $10^7$ yrs after emplacement but are restricted to a 1 to 2 km region around the top of the pluton (fig. 12). Paragenesis of these fractures is in many respects analogous to that within the pluton (fig. 16). Fractures caused by thermally induced pore-fluid pressure increases are the first to occur at any given location in the host rocks and are followed much later—on the order of $10^7$ yrs—by fractures caused by differential expansion of the bulk rock. However, the former type of failure extends for a much greater distance away from the pluton than the latter; the thermal anomaly dissipates before the thermally produced fracture region becomes extensive. The time lag in fracturing events will cause the resultant fractures to have systematic but diverse orientations, again reflecting temporal variations in the stress field. And, as in the pluton, these spatial and temporal variations in orientation will cause larger variations in the composition of aqueous solutions reaching the pluton, which in turn will result in diverse alteration mineralogy in the host rocks and pluton.

Though the model we have used is fairly simplified and specific, the conclusions are consistent with the limited geologic data available. More sophisticated approaches which afford for convective heat transfer, magma of different sizes, various emplacement conditions, and continuous temporal variations in material properties will serve mainly to refine results of the crystallization history and the timing, location, and duration of fracturing. For instance, convection within the host rocks and a larger magma body would cause fracturing to be initiated earlier and extend
over greater distances in the rocks above the pluton. A greater initial H₂O content within the magma would result in a large increase in magma pressure induced fracture events.

Future examinations of fracture evolution in hydrothermal systems must account for the concurrent mass transfer between fluid and rocks. Open through-going fractures will be conduits for circulating fluids, and in general, these fluids will be in chemical disequilibrium with the encasing wall rocks. Recent work (Helgeson, 1979) indicates that equilibrium will be rapidly attained — on the order of days — for geologically reasonable rates of advection and chemical affinity. Subsequent precipitation of reaction products on the walls of fractures will result in a slower — less than 10³ yrs (Knapp, 1977) — but still geologically rapid closure of the open fractures. But the ubiquitous occurrence of asperities along the fractures suggests that they will not close completely but will instead turn into a discontinuous sequence of isolated pores. This brings us full circle to fracture production by thermally induced pore-fluid pressure increases discussed previously; temperature increases will result in fracturing and be a drive toward linking the series of isolated pores and propagating the fracture into an open through-going crack again. The rate of this fracture propagation appears to be the slowest of all rates involved — strain rates are on the order of 10⁻¹³ sec (Norton and Knapp, 1979) — and thus is the rate limiting step. Solution saturated fractures in hydrothermal systems may be composed of a series of isolated pores for a major fraction of their history.

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REFERENCES


Shaw, H. R., 1965, Comments on viscosity, crystal settling, and convection on granite magmas: Am. Jour. Sci., v. 263, p. 120-152.


